**FINAL EXAMINATION**

*This is a closed book exam. You are only allowed to use class notes and lectures. You are not to consult with anyone or use Internet resources. Show all your work! I will take points off if you do not show me the derivations and all the steps.*

**QUESTION 1.** Consider the following game $G$:

(a) Derive the normal and fully reduced normal forms of $G$.

(b) Find all Nash equilibria.

(c) Assume that a player can never reasonably expect its opponent to choose a strictly dominated strategy, even off the equilibrium path. In this case, what is the unique perfect Bayesian equilibrium? (Make sure you show me the proof.)

**QUESTION 2.** A representative, $R$, has to decide how to vote on a bill before the House. Based on her understanding of the issues involved, she believes that the legislation is not important to the major interest group in her district and consequently she plans to oppose it. However, if the issue was important, she would prefer to support it. There is one interest group, $G$, that is known to favor the issue but $R$ is unsure whether the issue is of low priority to $G$ or of intense interest. $G$ itself knows how important the legislation is. Initially, $R$ believes that the issue has low priority for $G$ with probability $1/2$. The group can engage in two types of lobbying activities: a cheap campaign costs nothing and an expensive campaign costs $c > 0$. $R$ observes the choice of campaign by $G$ and decides which way to vote.

The payoffs are as follows: $R$ gets 1 if she supports a high-intensity issue or opposes a low-priority one; she gets 0 if she opposes a high-intensity issue or supports a low-priority one. If she opposes the bill, the interest group $G$ gets a payoff of 0 regardless of the intensity of interest. If she supports the bill, $G$ gets a payoff of 1 if the issue is of low priority and $v > 1$ if it is of great interest. The cheap campaign costs nothing but the expensive campaign costs $c > 0$ regardless of how much $G$ cares about the issue.
(a) Formulate this as an extensive form game of imperfect information and draw the game tree. Label everything carefully.

(b) Suppose \( c > 1 \). Find the perfect Bayesian equilibria (ignore knife-edge conditions on the exogenous variables).

(c) Suppose \( c < 1 \). Find the perfect Bayesian equilibria (ignore knife-edge conditions on the exogenous variables).

**Question 3.** Find the unique perfect Bayesian equilibrium in the following three-player game. Player 3’s information set includes all three of his decision nodes labeled 3.\( \alpha \), 3.\( \beta \), and 3.\( \gamma \). (Hint: let \( p \) denote player 3’s belief that he is at node 3.\( \beta \), and write Bayes rule for that.)