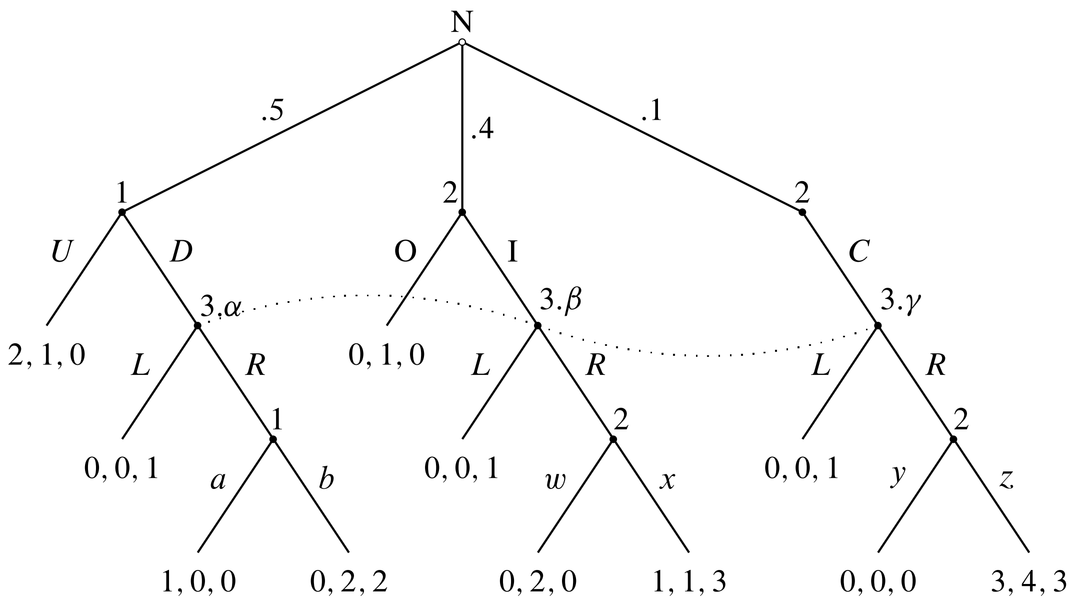
FINAL EXAMINATION

This is a closed book exam. You are only allowed to use class notes and lectures. You are not to consult with anyone or use Internet resources. ***Show all your work! I will take points off if you do not show me the derivations and all the steps.***

QUESTION 1. Find the unique perfect Bayesian equilibrium in the following three-player game. Player 3's information set includes all three of his decision nodes labeled $3.\alpha$, $3.\beta$, and $3.\gamma$. (Hint: let p denote player 3's belief that he is at node $3.\beta$, and write Bayes rule for that.)



QUESTION 2. THE CHAIN-STORE PARADOX. Wal-Mart operates stores in two towns, A and B, and faces potential competition from K-Mart, which decides sequentially whether it should build stores in these towns. Each time after observing K-Mart's action, Wal-Mart decides whether to acquiesce to an entry or start a price war. Assume that (i) if K-Mart stays out of town A, then it also stays out of town B, and (ii) if Wal-Mart acquiesces to entry in town A, then K-Mart always enters in town B and Wal-Mart acquiesces there as well. The payoffs are as follows: if K-Mart stays out of town A, the payoffs are $(2, 10)$, where the first payoff is K-Mart's and the second is Wal-Mart's; if it enters in A and Wal-Mart acquiesces, the payoffs are $(4, 4)$; if K-Mart stays out of town B, the payoffs are $(1, 5)$; if it enters and Wal-Mart fights, the payoffs are $(0, 0)$ and if Wal-Mart acquiesces, the payoffs are $(2, 2)$.

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(a) Write the extensive form of this game and find the subgame-perfect equilibrium. Write the strategic form of this game and find all pure-strategy Nash equilibria. Which of these survive iterated elimination of weakly dominated strategies?

(b) Assume now that unbeknownst to K-Mart, the management of Wal-Mart might get replaced by a robot that always fights entry no matter what. K-Mart estimates that the probability of this event is $\epsilon > 0$.

  (i) Write the strategic form and find all pure-strategy Nash equilibria. Which of these are subgame perfect? What happens as $\epsilon \to 0$?

  (ii) Write the extensive form and find the perfect Bayesian equilibria. What happens as $\epsilon \to 0$? Interpret the result.

QUESTION 3. REAL MEN DON'T EAT QUICHE. Player 1 is sitting in a bar and can choose to *drink beer* (B) or *eat quiche* (Q). He obtains 3 units of utility if he consumes his most preferred breakfast and 2 units if he consumes the stuff he does not like. Real men prefer beer and wimps prefer quiche. Player 2 walks into the bar and observes Player 1's choice of nourishment. Player 2 is the rowdy type and wants to pick a fight, and so has two actions, *fight* (F) and *not fight* (N). This player's payoff does not depend on what Player 1 is consuming but does depend on Player 1's type. Player 2 is a coward who really only wants to challenge a wimp. Thus, if Player 2 chooses to fight, he gets 1 if his opponent is a wimp, and 0 if his opponent is a real man. If, on the other hand, Player 2 chooses not to fight, he gets 0 if his opponent is a wimp, and 1 if his opponent is a real man. If a fight occurs, Player 1 loses 2 units of utility regardless of his type.

Suppose that the (common knowledge) prior is that Player 1 is a real man with probability 0.9. Draw the extensive form of this game and find the perfect Bayesian equilibria (be sure to specify any off-the-path beliefs where Bayes rule is undefined). Do any of them appear unreasonable? Why? (Hint: think about the conjectures player 2 must make to support one of the PBE, and then think what this implies for player 1's behavior.)