**QUESTIONS**

**Question 1.** The Pirates and the Spoils. There are $n$ pirates, all ranked according to a strict hierarchy with the first pirate being the Captain, the second being the next in command, and so on and so forth down to the very last pitiful crewman. For simplicity, assume pirate 1 is the captain, 2 is the next in command, and so on down to $n$, who is the lowly crewman.

Presently, the pirates capture a merchant ship and have to decide how to divide the spoils consisting of $m > n$ gold coins. The procedure they use is as follows. The highest ranking pirate proposes some distribution of the coins. If at least half of the pirates agree to the proposal, it is implemented and the game ends. If, however, more than half the pirates fail to agree, then the proposer is made to walk the plank to his death and the next highest ranking pirate becomes the proposer. The procedure is then repeated. Pirates are self-preserving, greedy, and vindictive in that order: they most prefer to live, then to get as much gold coins as possible, and given both of these, to see as many superiors walk the plank as possible.

What is your guess about the Captain’s predicament at the outset: is he in a strong position or is he screwed? (Write this down before solving the game.) What is the subgame perfect equilibrium of this game? How does this correspond to your prediction?

**Question 2.** Consider the following incumbent-challenger game, in which once a challenger enters, both candidates must decide to campaign in rural areas or urban areas.

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   Enter
   /    |
1  
0, 2  
1
   /    |
Urban
Rural
   /    |
Urban
Rural
   /    |
−6, −6
−1, 1
1, −1
−3, −3
```

Find all Nash equilibria. Which ones are subgame perfect?
**Question 3.** Two people take turns removing stones from a pile of \( n \) stones. Each person may, on each of his turns, remove either one stone or two stones. The person who takes the last stone is the winner and gets $1 from the other person. Player 1 gets to move first. Who is the winner in the SPE for an arbitrary \( n \)? Show all your work. (Hint: try solving the game for several small values of \( n \) and then prove the general result by induction.)

**Question 4.** There are two players, a buyer and a seller. The buyer’s value for the object is \( v > 0 \). Initially, the buyer chooses an investment level \( I \) that can be either high, \( I_H \), or low, \( I_L \), with \( I_H > I_L \). This increases the buyer’s value of the object to \( v + I \) but costs \( I^2 \). The seller does not observe the investment level and offers the object at a price \( p \). If the buyer accepts, his payoff is \( v + I - p - I^2 \), and the seller’s payoff is \( p \). If the buyer rejects, his payoff is \( -I^2 \), and the seller’s payoff is 0. Find the subgame perfect equilibria.

**Question 5.** Consider the following two-player game. Player 1 chooses whether to play the game, \( P \), or not, \( N \). If he chooses not to play, the game ends with payoffs \((1, 1)\). If he chooses to play, each player simultaneously announces a non-negative integer and his payoff is the product of these integers. Formulate this as an extensive form game and find its subgame perfect equilibria. (Hint: the game does have at least one SPE.)

**Question 6.** The Chain-Store Paradox. Wal-Mart operates stores in two towns, A and B, and faces potential competition from K-Mart, which decides sequentially whether it should build stores in these towns. Each time after observing K-Mart’s action, Wal-Mart decides whether to acquiesce to an entry or start a price war. Assume that (i) if K-Mart stays out of town A, then it also stays out of town B, and (ii) if Wal-Mart acquiesces to entry in town A, then K-Mart always enters in town B and Wal-Mart acquiesces there as well. The payoffs are as follows: if K-Mart stays out of town A, the payoffs are \((2, 10)\), where the first payoff is K-Mart’s and the second is Wal-Mart’s; if it enters in A and Wal-Mart acquiesces, the payoffs are \((4, 4)\); if K-Mart stays out of town B, the payoffs are \((1, 5)\); if it enters and K-Mart fights, the payoffs are \((0, 0)\) and if K-Mart acquiesces, the payoffs are \((2, 2)\).

(a) Write the extensive form of this game and find the subgame-perfect equilibrium. Write the strategic form of this game and find all pure-strategy Nash equilibria. Which of these survive iterated elimination of weakly dominated strategies?

(b) Assume players make small mistakes such that when they take some action, chance selects the action the player did not choose with a tiny probability \( \epsilon > 0 \) and the action the player did choose with probability \( 1 - \epsilon \). The mistakes are independent. (For example, if K-Mart chooses to enter in town A, then the game will continue with its entry with probability \( 1 - \epsilon \) but will continue as if it chose to stay out with probability \( \epsilon \).) Write the extensive form of this game and find the SPE. Write the
strategic form and find all PSNE. What happens as \( \epsilon \to 0 \)?

(c) Assume now that unbeknownst to K-Mart, the management of Wal-Mart might get replaced by a robot that always fights entry no matter what. K-Mart estimates that the probability of this event is tiny, \( \epsilon > 0 \). Write the extensive form and find the SPE. Write the strategic form and find all PSNE. What happens as \( \epsilon \to 0 \)?