GAMES OF INCOMPLETE INFORMATION

Show all steps and calculations in your answers.

**QUESTION 1.** Consider the two strategic-form games in Figure 1. Find the perfect Bayesian Nash equilibria in the following games:

(a) Nature chooses the game to the left with probability \( \pi \in (0, 1) \). Neither player observes the choice.

(b) Player 1 observes Nature’s choice but player 2 does not.

(Hint: construct the strategic forms.)

**Question 2.** Consider the following extensive form game.

(a) Find a subgame perfect equilibrium to this game. Is it unique? Are there any other Nash equilibria?

(b) Suppose that player 2 cannot observe player 1’s move. Write down the new extensive form. What is the set of Nash equilibria?

(c) Now suppose that player 2 observes player 1’s move correctly with probability \( p \in (0, 1) \) and incorrectly with probability \( 1 - p \). That is, if player 1 chooses \( U \), then player 2 observes \( U \) with probability \( p \) and observes \( D \) with probability \( 1 - p \).
Suppose that this facet of the situation and the value of \( p \) are common knowledge. What is the extensive form of this situation? Find all perfect Bayesian equilibria. (Hint: be very careful with the new extensive form; different actions by player 1 will lead to the same information set for player 2; think about what player 2 “sees” and how this relates to player 1’s true action. Write out Bayes rule first and derive the best responses. To search for equilibria from these best responses, consider pooling first, then separating, and finally semi-separating equilibria in which both types mix or exactly one type mixes. There are many equilibria in this game.)

**Question 3.** (Costly Signaling in Crises.) Consider the game in Figure 2. A defender, \( S_1 \), is trying to deter a challenger, \( S_2 \), from demanding a territory currently in his possession. He can make a costly threat, \( m \geq 0 \), that he will fight if she challenges. The challenger sees this threat and decides whether to initiate or not. If she does not challenge, the status quo obtains with the defender keeping the disputed territory, which he values at \( v_1 > 0 \), and the challenger getting nothing. If she does challenge, the defender decides whether to honor his threat. If he chooses not to fight, the challenger obtains the disputed territory, which she values at \( v_2 > 0 \), and the defender gets nothing. If he chooses to fight, the game ends in war, which the defender wins with probability \( p \in [\frac{1}{4}, \frac{3}{4}] \) and loses with probability \( 1 - p \). The winner obtains the territory, and war is costly for both. The costs are \( k_1 \in [1, 2] \) for the defender and \( k_2 \in [1, 2] \) for the challenger. The threat costs \( m \), which the defender must pay regardless of the outcome of the interaction. Assume that the true valuations are \( 0 < v_1 < 10 \).

![Figure 2: Costly Signaling.](image)

Find the **pure-strategy** perfect Bayesian equilibria for the following situations (ignore knife-edge scenarios where exogenous variables must have some exact values):
(a) The valuations of both players are common knowledge.

(b) The valuation of the challenger is common knowledge but only the defender knows his own valuation. The challenger believes that \( v_1 \) is drawn from the uniform distribution on the interval \([0, 10]\), and this belief is common knowledge. For this part, look for PBE in which only two threats are made, \( m = 0 \) and \( m^* > 0 \). (Hint: the PBE will have a cut-point property where \( S_1 \) will use \( m = 0 \) if his valuation is below a critical threshold called \( \hat{v}_1 \), and will use \( m^* \) if his valuation is above that threshold. You need to derive this cut-point value, which in turn will determine \( S_2 \)'s posterior beliefs.)