FINAL EXAMINATION

*Write your name and ID number on the front of the blue book. Do not forget to **sign the waiver on the back**. Answer both parts of the exam. Write legibly. Good luck!*

A. Answer both of the following two questions. Show all your work.

QUESTION 1. SOCIAL CHOICE. There are five individuals, A, B, C, D, and E, who have to select among five alternatives labeled v, w, x, y, and z. The individual preference orderings are

A	B	C	D	E
z	w	y	x	v
v	x	x	y	z
y	y	z	w	y
x	z	v	z	x
w	v	w	v	w

(a) Construct the social preference ordering under pairwise majority rule. Is it rational? Why or why not?

(b) Based on your answer in (a), do you think the individual preference orderings are single-peaked? Why?

(c) Plot the individual preference orderings such that they demonstrate your answer in (b). If they are single-peaked, who is the median voter and what would the Median Voter Theorem predict?

(d) Suppose now individual D's preference ordering is $x \succ z \succ w \succ y \succ v$, and the other four are the same as before. What happens to the social preference ordering?

(e) If the individual preference orderings are as in (d) and individual D is the agenda-setter, what would the social choice be? Explain why and how to obtain it.

Answer. (a) The social preference ordering is $y \succ x \succ z \succ v \succ w$. It is rational because it is complete and transitive.

(b) The individual preference orderings must be single-peaked because the group ordering is rational.

(c) The configuration of the ideal points is in Figure 1. The median voter is C, and the Median Voter Theorem would predict that his most preferred alternative y would win.

(d) The social ordering will be irrational because of the cycle $z \succ y \succ x \succ z$.

(e) D can ensure that x wins by putting y and z to a vote first, in which case $z \succ y$, and then x with z, in which case $x \succ z$. Afterward, x beats both v and w, and so it will be the social choice.

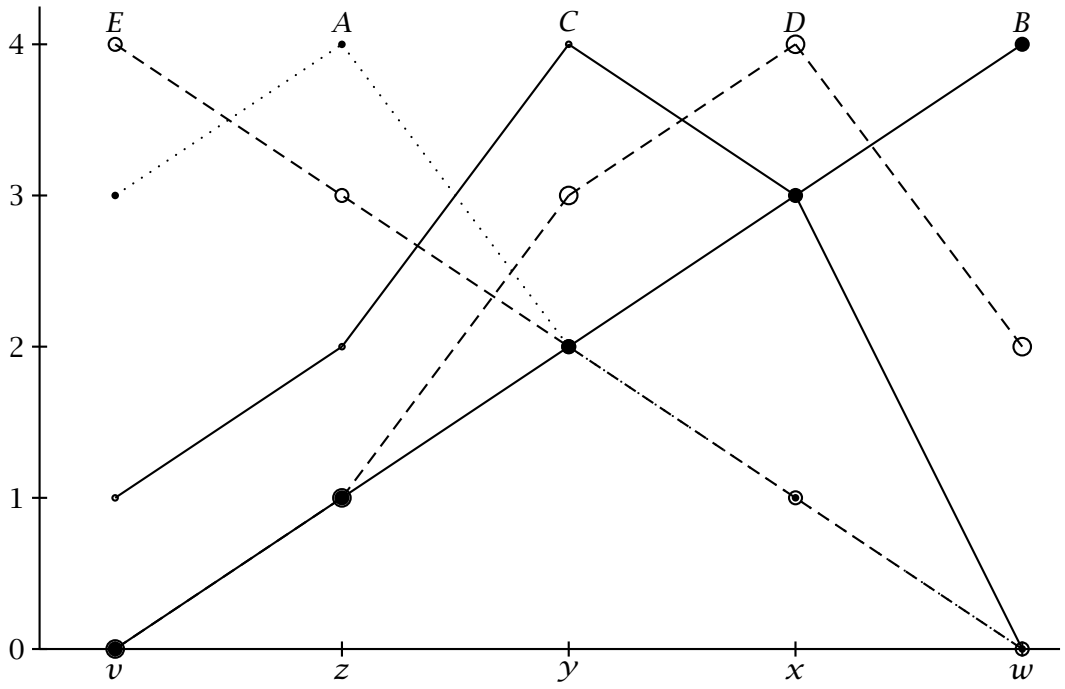

Figure 1: The Single-Peaked Preferences.

QUESTION 2. NUCLEAR PROLIFERATION. The United States and the Soviet Union are engaged in a game to decide whether to proliferate nuclear weapons. The United States can choose to give nukes to its NATO allies (D) or control them (C). The Soviet Union can choose to give nukes to its Warsaw Pact allies (D) or control them (C). The payoff matrix is represented in the figure below:

		USSR: Control (C)	USSR: Proliferate (D)
US: Control (C)		$(4,4)$ superpower nuclear monopoly, danger of escalation controlled	$(1,3)$ Warsaw Pact with nukes, NATO forces vulnerable
US: Proliferate (D)		$(3,1)$ NATO with nukes, Warsaw Pact forces vulnerable	$(2,2)$ NATO/Warsaw Pact go nuclear, danger of local escalation

Figure 2: The Proliferation Game.

(a) Find all pure-strategy Nash equilibria.

(b) Let p denote the probability that the US controls, and let q denote the probability that the USSR controls. Find the mixed-strategy Nash equilibrium. What are the expected payoffs in this equilibrium? Show all calculations.

(c) Suppose now the US moves first, followed by the Soviet Union. After each makes its proliferation decision, a crisis erupts between the US and a Soviet protege P. In the crisis, the US can issue a threat (T) or not issue a threat ($\neg T$). If no threat is issued, the protege gets its way, and if a threat is issued, the protege responds by either fighting (F) or not fighting ($\neg F$). Draw the extensive form of this new game. (Hint: you should get 12 outcomes.)

(d) Let r_{US} denote the US payoff from the proliferation game in Figure 2, and let r_{SU} denote the Soviet payoff. For example, if the US played C and the USSR played D, then $r_{US} = 1$, and $r_{SU} = 3$. Assign the following payoffs to the outcomes in (c):

- If the outcome is one of the four where no threat is issued, the payoffs are
  - US: $r_{US} - 2$
  - USSR: $r_{SU} + 1$
  - Protege with nukes: 3
  - Protege without nukes: 2

  For example, suppose that the US played C and the USSR played D (which means that P has nuclear weapons). Then, if the US plays $\neg T$, the payoffs would be
  - US: $r_{US}(CD) - 2 = 1 - 2 = -1$
  - USSR: $r_{SU}(CD) + 1 = 3 + 1 = 4$
  - Protege: 3 (because it has nuclear weapons)
- If the outcome is one of the four where the Protege capitulates (that is, the US plays T and P plays $\neg F$), the payoffs are
  - US: $r_{US} + 1$
  - USSR: $r_{SU} - 1$
  - Protege with nukes: 1
  - Protege without nukes: 0
- If the outcome is among the four war outcomes (the US chooses T, and P chooses F), then the payoffs are the expected utilities of war. War can end either in victory for the US or a defeat for the US. The probability that the US wins the war against a protege that does not have nuclear weapons is 0.8. The probability that the US wins the war against a protege that has nuclear weapons is only 0.1. The payoffs for victory and defeat are given in Table 1:

	US wins	US loses
US	$r_{US} + 3$	$r_{US} - 3$
USSR	$r_{SU} - 3$	$r_{SU} + 3$
Protege with nukes	-4	4
Protege without nukes	-3	3

Table 1: Payoffs from US victory and defeat in war.

Calculate each player's expected utility of war for each of the four possible outcomes where war occurs. For example, $EU_{US}(C, C)$ would denote the US expected utility for war if neither side proliferated in the initial stage of the game.

(e) Find the subgame perfect equilibrium (SPE) for the game in (d) by using backward induction. What is the SPE outcome?

Answer. (a) There are two pure-strategy Nash equilibria: (C, C) and (D, D).

(b) We use the payoff-equating method, as usual:

$$\begin{aligned} EU_{\text{US}}(C) &= 4q + 1(1-q) = 3q + 1 \\ EU_{\text{US}}(D) &= 3q + 2(1-q) = q + 2 \\ 3q + 1 &= q + 2 \\ 2q &= 1 \\ q &= \frac{1}{2} \end{aligned} \qquad \begin{aligned} EU_{\text{SU}}(C) &= 4p + 1(1-p) = 3p + 1 \\ EU_{\text{SU}}(D) &= 3p + 2(1-p) = p + 2 \\ 3p + 1 &= p + 2 \\ 2p &= 1 \\ p &= \frac{1}{2} \end{aligned}$$

So the mixed-strategy Nash equilibrium (MSNE) is $(1/2, 1/2)$. That is, both players choose C and D with equal probabilities of $1/2$. The expected payoffs are

$$EU_{\text{US}}(\text{MSNE}) = 4pq + 1p(1-q) + 3(1-p)q + 2(1-p)(1-q) = \frac{4}{4} + \frac{1}{4} + \frac{3}{4} + \frac{2}{4} = \frac{10}{4} = 2.5$$

$$EU_{\text{SU}}(\text{MSNE}) = 4pq + 3p(1-q) + 1(1-p)q + 2(1-p)(1-q) = \frac{4}{4} + \frac{3}{4} + \frac{1}{4} + \frac{2}{4} = \frac{10}{4} = 2.5$$

(c) The extensive-form is in the figure below (the payoffs are triples, ordered as US/USSR/P):

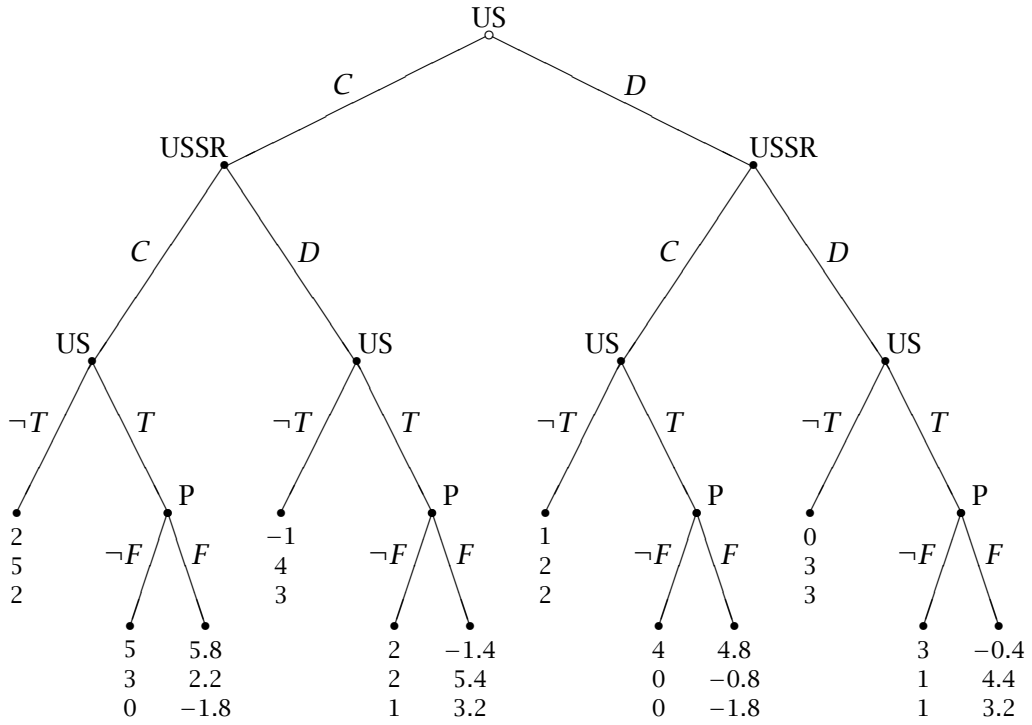

Figure 3: Proliferation and Crisis.

(d) The expected utilities of war are

$$EU_{US}(CD) = 0.1(1+3) + 0.9(1-3) = -1.4 \qquad EU_{SU}(CD) = 0.1(3-3) + 0.9(3+3) = 5.4$$
$$EU_{US}(DD) = 0.1(2+3) + 0.9(2-3) = -0.4 \qquad EU_{SU}(DD) = 0.1(2-3) + 0.9(2+3) = 4.4$$
$$EU_{US}(DC) = 0.8(3+3) + 0.2(3-3) = 4.8 \qquad EU_{SU}(DC) = 0.8(1-3) + 0.2(1+3) = -0.8$$
$$EU_{US}(CC) = 0.8(4+3) + 0.2(4-3) = 5.8 \qquad EU_{SU}(CC) = 0.8(4-3) + 0.2(4+3) = 2.2$$

$$EU_P(CD) = 0.1(-4) + 0.9(4) = 3.2$$
$$EU_P(DD) = 0.1(-4) + 0.9(4) = 3.2$$
$$EU_P(DC) = 0.8(-3) + 0.2(3) = -1.8$$
$$EU_P(CC) = 0.8(-3) + 0.2(3) = -1.8$$

(e) We begin the backward induction from the left, following (C, C, T), the protege would choose $\neg F$ because it yields it 0 versus -1.8. This means that the US would choose T, which gives it 5 while $\neg T$ only yields 2.

Following (C, D, T), the protege would choose F, which yields it 3.2, which is better than 1 from $\neg F$. Given this, the US would prefer $\neg T$ and get -1 to T which would yield -1.4.

Given the optimal strategies by the US and P, the Soviets would get 3 if they play C and 4 if they play D, so they would play D if the US chooses C.

Moving to the right side, after (D, C, T), the protege would choose $\neg F$ which yields 0 as opposed to getting -1.8 from F. Given this strategy, the US would play T, getting 4 instead of 1 from $\neg T$.

After (D, D, T), the protege would choose F, getting 3.2 versus only 1 from $\neg F$. Given this strategy, the US would choose $\neg T$ getting 0, versus T that would yield -0.4.

Given these optimal strategies, the USSR would choose D getting 3 instead of C which would net it only 0.

Given that the USSR would play D when it has the chance, the US strictly prefers to play D (getting 0) to C (getting -1).

The subgame perfect equilibrium is for the US to play D at the first node, then T following (C, C) or (D, C), and $\neg T$ following (C, D) or (D, D); the USSR to play D regardless of what the US plays in its first move; and P to play $\neg F$ following (C, C, T) or (D, C, T), and F following (C, D, T) or (D, D, T).

The subgame-perfect equilibrium outcome is for both US and USSR to proliferate, and the US to back down in the crisis: $(D, D, \neg T)$.

B. Answer <u>any five</u> of the following six questions.

QUESTION 1. Recall that "extended deterrence" refers to situations where a defender pledges to defend a protege against encroachments by some potential challenger. Suppose you read a study that claims that in 80% of extended deterrence cases, the defender fails to protect its protege when challenged. The study concludes that extended deterrence is useless because it fails too often. Discuss.

Answer. This is the standard problem of **selection bias**. Only cases where the challenger believes that the defender's commitment is not strong enough would appear in the history record as failed cases of extended deterrence. However, since the challenger does not attack a protege when the defender's credibility is not an issue, we cannot know how often extended deterrence actually works in practice (because the challenger may have been deterred by the threat or may not have attacked even without the threat). The conclusion is that the study is wrong in making the claim (although we cannot say that extended deterrence actually works either).

QUESTION 2. What are the two rationalist explanations of war? Discuss how they demonstrate the possibility of bargaining failure.

Answer. The two explanations are **private information with incentives to misrepresent** and **dynamic commitment problems**. Players who have private information about factors that affect the outcome of war may fail to locate a mutually acceptable bargain because they can be both too optimistic about their expected utility of fighting and so may demand too much. Since they have incentives to misrepresent their information to gain bargaining advantages, communication will not be useful in revealing this information (cheap talk). In the other explanation, players may not be able to credibly commit to uphold the agreement in the future because of improved bargaining position that may allow one player to demand more from the other. Also, if there exist serious enough offensive advantages, players may not be able to commit not to strike first.

QUESTION 3. Explain the principal-agent problem and how it may generate the gamble for resurrection. Give at least one historical example discussed in class where the gamble may have occurred.

Answer. The office-seeking leader is the agent of the selectorate, the policy-motivated principal. The problem is that the preferences of the principal and the agent may not be aligned. If the leader is losing a war and faces a high probability of being removed from office, she may attempt to improve its electoral chances by fighting a war. The Falklands War is a prime example, in which both sides gambled for resurrection but only Thatcher was resurrected.

QUESTION 4. What is the fundamental problem that mutually assured destruction (MAD) posed for deterrence? What strategies were suggested to overcome it? You may use the Cuban Missile Crisis to illustrate your argument.

Answer. The fundamental problem was that of credibility: under MAD no side can credibly threaten with nuclear weapons because the other side's second-strike capability would ensure that the other side could deliver a devastating retaliatory strike even after absorbing a first strike. The **threat that leaves something to chance** (a strategy of manipulating risk) can be used to increase credibility. **Relinquishing choice** is another strategy which the US successfully pursued in the missile crisis of 1962 because it saddles the other side with the decision to escalate the crisis.

QUESTION 5. Explain how cooperation may spontaneously emerge under anarchy. What are some of the problems that have to be overcome to sustain it?

Answer. Cooperation may emerge spontaneously under anarchy when interaction is repeated over a long period of time and when players care sufficiently about the future. Cooperation is sustained by using **conditional strategies** that punish defection. When players care enough about gains from cooperation in the future that they would lose by defecting today, they are deterred from defection. Players have to overcome problems with monitoring, enforcement, and multiple equilibria. **Transaction costs** account for difficulties with arriving at cooperative arrangements.

QUESTION 6. What are the two types of **audience costs** and how do they differ from **sunk costs**? Explain how audience costs can affect the credibility of commitments in alliances and during crisis bargaining.

Answer. The two types of audience costs are **external** and **domestic**. The first is generated by third parties that observe the actor's behavior and who change their beliefs/expectations based on information revealed by this behavior. The second is generated by the domestic voters who may punish leaders for backing down after escalating a crisis or failing to follow through on promises. Audience costs differ from sunk costs in that they are generated during a crisis and are paid only if the leader fails to uphold her commitments. Sunk costs are paid before the crisis regardless of its outcome. Audience costs improve the credibility of alliances by increasing the probability that a defender would actually help its ally because if it does not (a) third parties may conclude its interests are not that strong and may challenge them also, and (b) domestic audiences may punish the leader for reneging. In crisis bargaining, domestic audiences could help a leader commit to not backing down once they exceed the costs of fighting.