MID-TERM EXAMINATION

*Write your name and ID number on the front of the blue book. Do not forget to **sign the waiver on the back**. Answer both parts of the exam. Write legibly. Good luck!*

A. Answer both of the following two questions. Show all your work.

QUESTION 1. Consider the following game:

		Player 2 L	Player 2 R
Player 1	U	2, 4	0, 0
	D	1, 6	3, 7

(a) Find all pure-strategy Nash equilibria.

(b) Find the mixed strategy Nash equilibrium.

(c) Compute the players' expected utilities in the mixed strategy equilibrium.

(d) What is the probability of the worst outcome in the mixed strategy equilibrium?

Answer. Students must show their work!

(a) There are two Nash equilibria in pure strategies: (U, L) and (D, R). In these strategy profiles both players are playing best-response strategies. Neither can gain by deviating to another strategy given what its opponent is doing.

(b) Let p be the probability that Player 1 chooses U, and let q be the probability that Player 2 chooses L. We use the payoff-equating method:

$$\begin{aligned} EU_1(U) &= EU_1(D) \\ 2q + 0(1-q) &= 1q + 3(1-q) \\ 4q &= 3 \\ q &= \frac{3}{4} \end{aligned} \qquad \begin{aligned} EU_2(L) &= EU_2(R) \\ 4p + 6(1-p) &= 0p + 7(1-p) \\ 5p &= 1 \\ p &= \frac{1}{5} \end{aligned}$$

Therefore, in the mixed-strategy Nash equilibrium Player 1 chooses U probability $1/5$, and D with probability $4/5$; Player 2 chooses L with probability $3/4$, and R with probability $1/4$.

(c) In the mixed-strategy equilibrium, the expected utilities of both actions are the same, so we only need to calculate the expected utility for one action for each player:

$$EU_1(U) = 2q + 0(1-q) = 2q = 2(0.75) = 1.5$$
$$EU_2(R) = 0p + 7(1-p) = 7 - 7p = 7 - 7(0.2) = 5.6$$

(d) The worst outcome is (U, R) where both players receive zero. The probability of that outcome is

$$\Pr(U, R) = \Pr(1 \text{ plays } U) \times \Pr(2 \text{ plays } R) = p \times (1-q) = 0.2 \times 0.25 = 0.05$$

Question 2. The Envelope Game. There are two players, player 1 and player 2, and two envelopes, one of which is marked for player 1, and the other is marked for player 2. At the beginning of the game, each envelope contains one dollar. Player 1 is given the choice between stopping, S, or continuing, C. If she chooses to stop, then each player receives the money in her own envelope and the game ends. If she chooses to continue, then one dollar is removed from her envelope and two dollars are added to player 2's envelope. This completes the first round of the game. Then player 2 must choose whether to stop or continue. If she chooses to stop, each player gets the money in her envelope. If she chooses to continue, then one dollar is removed from her envelope and two dollars are added to player 1's envelope. This completes the second round of the game. The game continues like this alternating between the players until either one of them decides to stop or k rounds of play have elapsed. If neither player chooses to stop by the end of the kth round, then both players obtain zero. Players want to earn as much money as possible.

(a) Draw the extensive form of this game for $k = 5$.

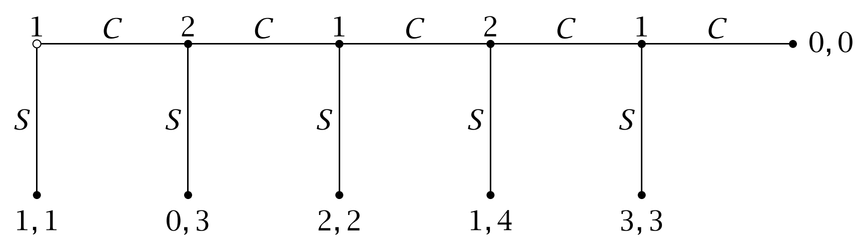
(b) Find the subgame perfect equilibrium by using backward induction.

(c) What is the subgame perfect equilibrium outcome?

(d) What do you think the backward induction outcome would be for any finite integer k? Why do you think this outcome obtains?

Answer. This is actually a famous game often used to illustrate the shortcomings of backward induction!

(a) Here's the game tree for $k = 5$:



(b) It is easy to see that each player's optimal move in every round is to stop. In the last round, stopping yields player 1 \$3 while continuing yields zero. So, S is optimal there. Given this, player 2's payoff to C is \$3, while stopping yields \$4, so she chooses to stop. Given this, player 1's payoff in $k = 3$ from C is \$1 and her payoff from S is \$2, so she stops. Given that, player 2 would stop in $k = 2$, which means that player 1 would stop also in $k = 1$. The subgame perfect equilibrium is therefore the profile of strategies where both players always stop: (S, S, S) for player 1, and (S, S) for player 2.

(c) The outcome is that player 1 stops immediately and each player receives \$1.

(d) The same as in (c). Even though both players would be better off if they could play the game for several rounds, neither can credibly commit to not stopping when given a chance, and so they both end up with small payoffs.

B. Answer <u>any five</u> of the following six questions.

Question 1. What two properties must an actor's preference ordering satisfy to be considered rational? What do these properties require?

Answer. The preference ordering must be **complete** and **transitive**. Completeness requires that the actor can rank order all available alternatives with respect to each other. Transitivity requires that if he prefers A to B and B to C, then he also prefers A to C for all A, B, and C.

Question 2. In class it was argued that a theory must satisfy three requirements, one of which was **empirical validity**. What are the other two and what do they mean? How can we use equilibrium analysis to judge a theory?

Answer. The other two are **logical consistency** and **falsifiability**. The first requires that (i) the various assumptions do not contradict each other and (ii) conclusions follow from the premises in a logically coherent way. The second requires that we can imagine a set of circumstances that would invalidate the theory's claims. We can use equilibrium analysis to judge the logical consistency of a theory and make predictions, which we can then test with statistical analysis or case studies.

Question 3. In class it was argued that the assumption of **anarchy** really has two components. What are they and what do they imply for strategic behavior? How can we address them with formal models?

Answer. The two components of anarchy are (i) the lack of central authority to enforce agreements, and (ii) the possibility to use force. They imply that all agreements must be self-enforcing and that actors may choose to use force (or threats to use force) at any time if they want to. We can incorporate the option to use force in our formal models, which we can then solve for subgame perfect equilibria that ensure that all threats and promises are credible. That is, equilibrium agreements are self-enforcing.

Question 4. What problems do players face in games of pure cooperation? Games of pure conflict (zero-sum games)? Games of distributional conflict (mixed-motive games)? What methods may they use to address these problems?

Answer. In games of pure cooperation players may face coordination problems. They can use tacit or explicit communication to coordinate on a focal point. In games of pure conflict players want to make sure the other cannot guess their action. They may use mixed strategies to do that. In distributional conflict games players want to reach an agreement but disagree over the terms of the agreement. They will use bargaining and strategic moves to influence each other's expectations.

Question 5. Bargaining is the process of influencing expectations and has two functions. What are they? What conditions must they meet to influence expectations successfully? Why?

Answer. The two functions are **transmission of information** and **establishment of commitments**. The first occurs when players signal information they know and screen the opponent for information he knows. Signals must be costly in order to influence expectations. If they are not (cheap talk), then any player can make any pronouncement and there will be no reason to believe them. Commitments are pledges to take actions in the future and must be understood and credible. If they are not communicated properly and understood, then they are useless because they other player cannot react to them even if he would have wanted to had he known about them. If they are not credible, then the other player will not believe them and his expectations will not change in our favor.

Question 6. We discussed two basic ways of establishing credible commitments, one of which depends on strategic moves and the other on resolve and nerve. We also discussed two strategies for each of these ways. What are these ways and what are the strategies? How do they work to increase credibility?

Answer. The two basic ways are **leaving the last clear chance to avoid disaster to the opponent** and **risking undesired and unintended consequences**. The two strategies for the first method are (a) to constrain our choices: eliminate an option that we might be tempted to use in the future, and (b) relinquish the initiative to the opponent: leave the painful choice to them. The two strategies for the second method are (a) take an action that leaves something to chance: pull the opponent to the brink and depend on him being less risk-acceptant and resolve so he pulls back first, and (b) use limited retaliation: increase current costs and the probability of inflicting future ones by not destroying everything the opponent values immediately.