PREFERENCES AND EXPECTED UTILITY

Write your answers neatly on a separate sheet of paper (attach as many sheets as necessary). Show all calculations (no credit for answers giving final result only). Justify your steps. Remember to write your name and staple all pages together.

QUESTION 1. THEORIES

Evaluate the following statements as True or False and explain very briefly (one sentence) your choice:

(a) We only need to know the facts to adequately explain any event.

(b) Theories are comprehensive and precise descriptions of reality.

(c) If a logically inconsistent theory makes correct predictions, then it is still useful.

(d) To evaluate a theory, we must establish the truthfulness of its assumptions.

Answer. Don't forget to give the reasoning.

(a) False: knowledge of facts is insufficient. There is selection bias (choosing which facts to consider), and implicit theoretical perspective (thinking how the different variables interact to produce consequences).

(b) False: theories are simplified descriptions of reality because reality is much too complex to define precisely.

(c) False: a logically inconsistent theory allows us to make almost any claim.

(d) False: a theory is logically consistent if its predictions follow from its assumptions and it is useful if it helps us understand the phenomena that it explains. How realistic an assumption is has no bearing on our judgment of theory—if an assumption is useful, then it is "correct."

QUESTION 2. "No war ever begins without a prior crisis. Some crises can be settled through bargaining, others cannot. If both states are democratic, they are always able to find a bargaining solution, but if both are communist they only can if neither believes it can win a war. A strong communist state always goes to war against a weak opponent, but only goes to war against a strong opponent if it believes it can win. In fact, if a state is reasonably confident in its ability to win, the crisis results in war whenever a bargaining solution cannot be reached." According to this claim, are the following necessary conditions, sufficient conditions, or both, for war to occur? Explain why.

(a) Presence of a crisis

(b) At least one state is not a democracy

(c) A crisis between a strong communist state and a weak adversary

(d) A crisis between two strong communist states

(e) A crisis where at least one state is communist and either its opponent is weak or it believes it can win.

Answer. Don't forget to give the explanations!

(a) Necessary; not sufficient because it does not always result in war.

(b) Necessary; not sufficient because bargaining solution still possible.

(c) Sufficient (war inevitably occurs); not necessary because war possible with other states (e.g. two strong communist states where one believes in victory).

(d) Neither: we must know the belief about victory.

(e) Both: necessary to have a non-democracy and sufficient for the communist to fight (always against a weak opponent and always when it believes it can win).

QUESTION 3. THE RATIONAL DICTATOR?

It's 1968. For various reasons, the dictator of a small, poor country wants to increase his nation's wealth. He sees before him the following options:

- S = become a client of the Soviet Union
- U = become a client of the United States
- D = begin growing cocaine and opium for foreign markets
- I = begin exporting ivory

Identify **why** the dictator is or is not rational (as defined in class) if his preferences are defined as in the following two cases:

(a) $S \succ U,\ U \succ D,\ S \succ D,\ D \sim I,\ U \succ I,\ S \sim I$

(b) $U \succ S,\ U \succ D,\ D \succ I,\ S \sim D,\ U \succ I$

(c) $S \sim U,\ U \sim D,\ D \succ I,\ S \sim D,\ U \succ I,\ S \succ I$

Answer. The answers should include the reasoning:

(a) intransitive: $S \sim I$ contradicts $S \succ D$ and $D \sim I$, which imply $S \succ I$

(b) incomplete: no preference between S and I

(c) rational: both complete and transitive

QUESTION 4. THE CIA DECISION CALCULUS

Unbeknownst to the dictator in the above problem, the CIA has decided to support one of two dissident factions in the country. However, the CIA is not completely certain

what option either of the factions would choose should they come to power. The CIA's (cardinal) utilities for the four outcomes are: $u(S) = 0,\ u(U) = 10,\ u(D) = .2,\ u(I) = 7$. The probabilities that the CIA assigns to the outcomes for each faction are:

- Marxists: $p(S) = .5,\ p(U) = .01,\ p(D) = .4,\ p(I) = .09$
- Nationalists: $q(S) = .05,\ q(U) = .4,\ q(D) = .4,\ q(I) = .15$

Which faction does the CIA prefer (show all calculations)?

Answer. We find the expected utilities for the two actions:

CIA's expected utility from supporting the Marxists is

$$\begin{aligned} EU(\text{Marxists}) &= p(S)u(S) + p(U)u(U) + p(D)u(D) + p(I)u(I) \\ &= (.5)(0) + (.01)(10) + (.4)(.2) + (.09)(7) \\ &= .81 \end{aligned}$$

CIA's expected utility from supporting the Nationalists is

$$\begin{aligned} EU(\text{Nationalists}) &= q(S)u(S) + q(U)u(U) + q(D)u(D) + q(I)u(I) \\ &= (.05)(0) + (.4)(10) + (.4)(.2) + (.15)(7) \\ &= 5.13 \end{aligned}$$

Since $EU(\text{Marxists}) < EU(\text{Nationalists})$, the CIA would support the Nationalists.

QUESTION 5. CRISIS BARGAINING

Imagine a state which is involved in negotiations with another state. There are three options available to it, which are

- S = stop the negotiations, in which case the situation remains unchanged—the *status quo* prevails;
- A = attack the other state, in which case the first one will win (W) with some probability p and lose (L) with probability $1 - p$;
- B = bargain for another round, in which case the other state may attack with some probability q. If the other state attacks, war occurs, and the first state can win it with probability p (i.e. there are no advantages to offense or defense). If the other state does not attack, the *status quo* is revised to the first state's advantage (C).

Let's assume the following utilities for the first state: $u(S) = 5,\ u(W) = 6,\ u(L) = 0,\ u(C) = 10$ (the last term is the utility of bargaining when the second state decides not to attack, i.e. it capitulates).

(a) Express the first state's expected utility of attacking immediately (the war lottery)

(b) Express the first state's expected utility of bargaining

(c) Given the utilities above and assuming the second state will attack with probability $q = .5$, what is the probability of winning a war that will make the first state prefer to attack immediately? Hint: use the formulae you constructed for (a) and (b).

Answer. The answers must show the calculations

(a) $EU(A) = pu(W) + (1 - p)u(L)$

(b) $EU(B) = qEU(A) + (1 - q)u(C)$

(c) We must find p such that $EU(A) > EU(S)$ and $EU(A) > EU(B)$. From (a), we obtain $EU(A) = 6p$ and from (b) we obtain $EU(B) = 3p + 5$. Since $3p + 5 > U(S) = 5$, we only need to consider $EU(A) > EU(B)$. This obtains whenever $p > \frac{5}{3}$. However, this is not a valid probability. Therefore, we conclude that there is no probability of winning a war that will make the state choose to attack. In other words, the state will never attack first.

QUESTION 6. COLLEGE CHOICE

Before going to college, you made an expected utility calculation about where to send your application, and then once you were admitted, where to choose to attend. Assume you are deciding to apply to one of three universities: Harvard, UCSD, and East Appalachian State:

- The utility of going to Harvard would be great. You would graduate and expect to get a high-paying job worth $80,000 annually. However, knowing your school record, you assess the probability that you would (1) be admitted, and (2) graduate with a decent record, to be fairly low—only 30%. That is, there is a 70% chance that you will end up with nothing.
- The utility of going to UCSD is good. Graduates makes a good ($40,000) salary on average, and you think it is likely (75%) that you will be admitted and will graduate with a good record.
- The utility of going to East Appalachian State is not so good. Graduates earn about $20,000 on average. Its advantage is that you are fairly certain (95%) that you will be admitted and will graduate successfully.

Assume that you can only apply to one college at a time. If you fail to get in or graduate with a good record, you end up with a $8,000 job at MacDonald's. Given this situation,

(a) Compute the expected utility for each option (show your work).

(b) Which college is the best rational choice in this situation? Why?

(c) What is the critical value of the probability of successful completion that would make choosing Harvard rational?

Answer. Let p_i be the probability of successful completion (admission and good record) for college i, $u(i)$, the utility of successful completion, and $u(M)$ the utility of working at MacDonald's.

(a) $EU(i) = p_i \times u(i) + (1 - p_i)u(M)$, and so:

- $EU(H) = p_H u(H) + (1 - p_H)u(M) = (.30)(80,000) + (.70)(8,000) = 24,000 + 5,600 = 29,600$
- $EU(U) = p_U u(U) + (1 - p_U)u(M) = (.75)(40,000) + (.25)(8,000) = 30,000 + 2,000 = 32,000$
- $EU(A) = p_A u(A) + (1 - p_A)u(M) = (.95)(20,000) + (.05)(8,000) = 19,000 + 400 = 19,400$

(b) You would go to UCSD (don't you feel smart now!) because it has the highest expected utility.

(c) We must solve $EU(H) \geq EU(U)$:

$$\begin{aligned} p_H u(H) + (1 - p_H)u(M) &\geq EU(U) \\ p_H u(H) + u(M) - p_H u(M) &\geq EU(U) \\ p_H[u(H) - u(M)] &\geq EU(U) - u(M) \end{aligned}$$

solving for p_H, we get

$$p_H \geq \frac{EU(U) - u(M)}{u(H) - u(M)}$$

substituting the numbers, we get

$$p_H \geq \frac{32,000 - 8,000}{80,000 - 8,000} = \frac{24,000}{72,000} = \frac{1}{3}$$

That is, the critical value is $p_H = 1/3$, or approximately .33 percent (which is why the 30% given above was insufficient). If you estimate your chances of successful completion at Harvard to be at least $1/3$, you should go for it.