SOCIAL CHOICE.
THE PRINCIPAL-AGENT PROBLEM

Write your answers neatly on a separate sheet of paper (attach as many sheets as necessary). Show all calculations (no credit for answers giving final result only). Justify your steps. Remember to write your name and staple all pages together.

QUESTION 1. Suppose there are seven individuals with the following preferences over three alternatives:

1	2	3	4	5	6	7
z	z	x	x	y	y	y
x	x	y	y	x	x	x
y	y	z	z	z	z	z

(a) What is the social preference ordering under pairwise majority voting?

(b) Is it rational? Why or why not?

(c) Instead of pairwise majority voting, consider *plurality voting* where each individual votes only for his most preferred alternative and alternatives are ranked socially by the total number of votes each receives. What is the social preference ordering now?

(d) Consider z to be the "irrelevant alternative", show, by changing the preferences of individuals 1 and 2 only, that plurality voting violates the independence of irrelevant alternatives condition. (That is, the social ranking of x and y does not depend only on each individual's ranking of x and y but also on the ranking of x and z or y and z. Hint: move alternative z to the bottom of the first two individuals' preference orderings.)

(e) Consider now the preference aggregation rule known as the *Borda count* where each individual casts a different number of votes depending on how he ranks the alternatives. Each individual casts 5 votes for his most preferred alternative, 4 votes for his second-best, and 1 vote for the least preferred one (that is, each individual has 10 votes). The votes for each alternative are then summed across individuals and the alternatives ranked according to these sums. What is the social preference ordering now?

(f) Construct a change in individuals 6 and 7's preferences such that there is a violation of the IIA condition.

Answer. (a) Four individuals prefer x to y, and so $x \succ y$, five individuals prefer y to z, and so $y \succ z$, five individuals prefer x to z, and so $x \succ z$. The social preference ordering is $x \succ y \succ z$.

(b) It is both complete and transitive, and therefore rational.

(c) y gets 3 votes, x and z each get 2. So the social preference ordering is $y \succ x \sim z$.

(d) Suppose we move z to the bottom, and so both 1's and 2's preference orderings become $x \succ y \succ z$. This preserves the ranking $x \succ y$ for these individuals. Alternative x now gets 4 votes and the social preference ordering is $x \succ y \succ z$. So, the social preference ordering changed from $y \succ x$ to $x \succ y$ even though the individual rankings of these two alternatives remained the same. Plurality rule violates IIA.

(e) Alternative x gets 30 votes, y gets 25, and z gets 15. The social preference ordering is therefore $x \succ y \succ z$.

(f) Change players 6 and 7's preference orderings to $y \succ z \succ x$. This change preserves their original ranking $y \succ x$. Alternative y still receives 25 votes, but now z gets 21 and x gets 24. The social preference ordering is $y \succ x \succ z$. So, even though the individual rankings of x and y remained the same, the social ranking switched from $x \succ y$ to $y \succ x$. Borda count violates IIA.

QUESTION 2. There are three individuals with the following preferences over three alternatives:

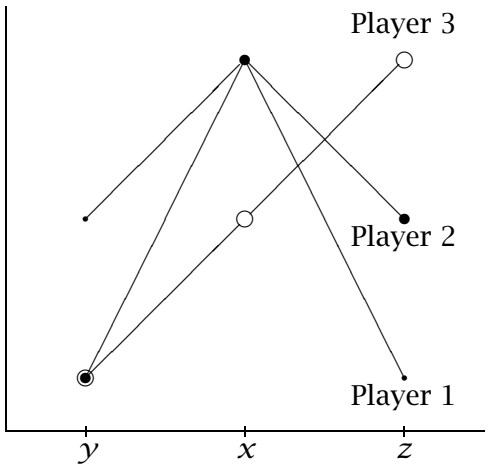
1	2	3
x	x	z
y	z	x
z	y	y

(a) Construct the social preference ordering under majority rule. Is it rational?

(b) Are these preferences single-peaked? (Hint: try putting the three alternatives on the horizontal axis in some order, say x, followed by y, followed by z, and see if changing this order changes the way preferences "peak". The answer in (a) should also clue you in about how this one must turn out.)

Answer. (a) All three prefer x to y, and so $x \succ y$; two prefer z to y, and so $z \succ y$; two prefer x to z, and so $x \succ z$. The social preference ordering is $x \succ z \succ y$. It complete and transitive, and therefore rational.

(b) From (a) we know that the preferences must be single-peaked. One way of showing single-peakedness is demonstrated below. (Another acceptable ordering along the horizontal axis would be z, x, y.) There is no guesswork here: you know from the Median Voter Theorem that x must be in the middle because it is the winning alternative.



QUESTION 3. NUCLEAR ARMS CONTROL. Figure 1 represents a negotiation between Peaceniks and Hawks over arms control. We want to reach an agreement that is acceptable to both. The principal appoints three agents who come from the groups "Students against the Bomb" (SAB), "Scholars for Responsible Defense" (SRD), and "Citizens for World Domination" (CWD). The range of options is from bilateral disarmament to no arms control whatsoever. Some of the intermediate positions (deep cuts, nuclear freeze, and selective limits) are shown as well. The status quo is at the selective limits (ABM, neutron bomb).

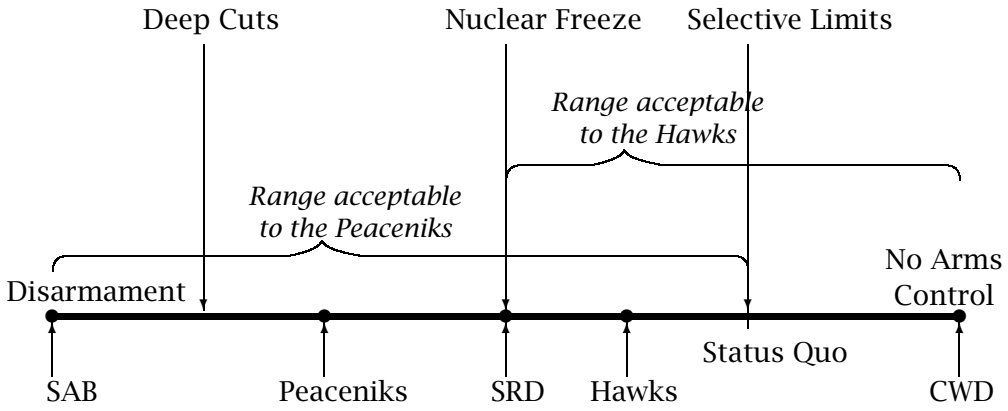


Figure 1: Nuclear Arms Control.

Referring to the ideal points of the three agents, the Peaceniks, and the Hawks, and the acceptance ranges for the two main actors as specified in Figure 1, answer the following questions:

(a) What will each agent offer? (Mark the positions on the figure.)

(b) Why do some agents propose their ideal points while others do not?

(c) Given the recommendations, where will the final deal fall, and why?

Answer. (a) SAB offers Nuclear Freeze. It cannot offer either Disarmament (its ideal point) or even Deep Cuts because both would be rejected by the Hawks. SRD offers its ideal point, Nuclear Freeze, which is acceptable to both the Peaceniks and the Hawks. CWD offers Selective Limits because its ideal point of No Arms Control would be rejected by the Peaceniks.

(b) Explained in (a).

(c) The final deal will fall somewhere between Nuclear Freeze and Selective Limits, which are the most extreme recommendations offered to the principal, and they are both acceptable. Anything in between falls into the bargaining range. It is very likely that the Hawks will obtain their ideal point because the Peaceniks would accept it over the Status Quo because it yields a higher utility, and because any other offer in the bargaining range yields the Hawks a lower utility.

QUESTION 4. INTERNATIONAL BARGAINING WITH DOMESTIC CONSTRAINTS. The executives of two countries are engaged in negotiating a bilateral trade treaty. The terms of trade can vary anywhere from autarky (no trade, high protectionist tariffs) to unfettered free trade. The status quo is autarky because no current trade relationship exists. The Executive of one of the states is constrained by a domestic ratification procedure that requires that any agreement between him and the leader of the Foreign state must be ratified by Congress. For an agreement to be

ratified, the median voter in Congress must agree to it. If the agreement fails to be ratified, the status quo prevails. The leader of the Foreign state is not constrained.

Figure 2 shows the ideal points of the two leaders, the location of the status quo and the range of acceptable bargains that they are prepared to strike. The overlap of the two leaders' acceptance regions represents the *bargaining range*; that is, this is the range where any possible agreement must have to lie in. Assume that everything is common knowledge. That is, all players involved know everything about each other.

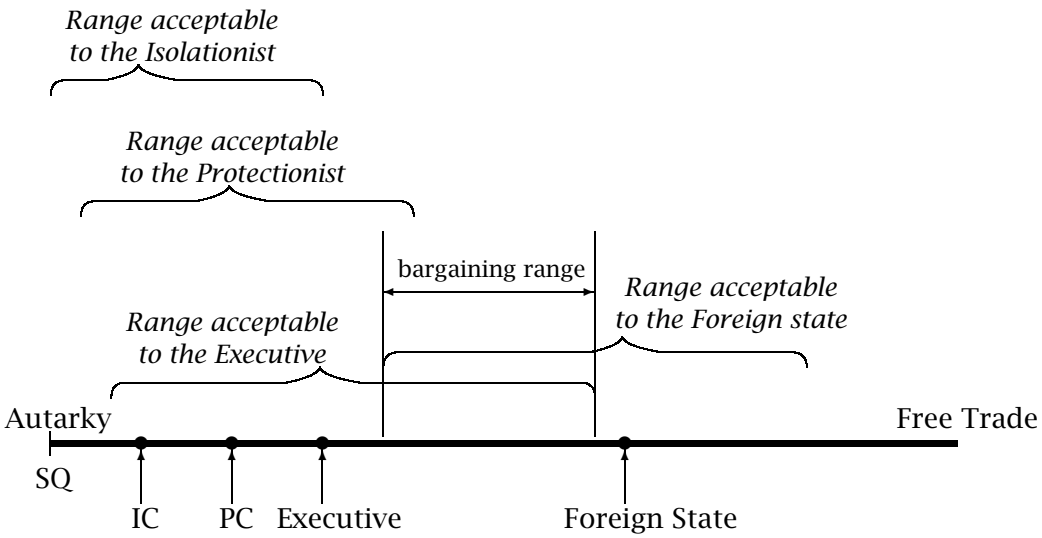


Figure 2: Bilateral Trade Treaty Negotiations.

(a) Suppose that the median voter in Congress has the same preferences as the Executive. What is the bargaining range now?

(b) Suppose that the median voter in Congress is a Protectionist (PC) with the acceptance range indicated in the figure. What is the bargaining range now?

(c) Suppose that the median voter in Congress is an Isolationist (IC) with the acceptance range indicated in the figure. What is the bargaining range now?

(d) How constrained is the Executive if the median voter has the same preferences? If he is a Protectionist? If he is an Isolationist? What can you say about the possible deals the Executive can reach given these different degrees of constraint. Can weakness (that is, being constrained by a domestic ratification procedure) be a bargaining strength in international negotiations? How does this relate to our findings about commitment by constraining one's available choices?

Answer. (a) The bargaining range remains the same. The executive is unconstrained, and the agreement will fall somewhere in that range.

(b) The new bargaining range consists of the intersection of PC's acceptance range and the original bargaining range. The executive is constrained because he cannot accept any offers to the right of the maximum offer accepted by PC. The bargaining range shrinks substantially.

(c) There is no bargaining range. The maximum offer that IC would accept is to the left of the minimal offer that the Foreign State would accept. Therefore, no deal is possible. The executive is extremely constrained.

(d) For the constraints, see the answers to (a), (b), and (c). A weaker executive (e.g. the one constrained by a protectionist Congress) will probably obtain a deal that will be closer to his ideal point because he can credibly commit not to offer any deals in the original bargaining range that would fall to the right of PC's acceptable maximum. He was unable to commit credibly not to offer these deals without the constraint because they all fall within his acceptance range. On the other hand, too much weakness (e.g. the constraint by an isolationist Congress) could be a problem because the executive will be unable to negotiate any deals. This is what happened to President Wilson when Congress refused to ratify American membership in the League of Nations.