Best Responses and Equilibrium

1 The Tariff Game

There are 2 players, player 1 and player 2, and each of them has two possible actions: put up a tariff barrier, T, or remove it and opt for free trade, F. Each players can export \$10 million worth of goods. However, if his partner imposes a tariff, his goods become less competitive and so he can sell only \$8 million worth of them. Supposing that the tariff imposes a 25% tax on these goods, the other player would obtain \$2 million in tax income from the tariff, reducing player 1's payoff to \$6 million. In this situation, player 1's payoff is \$6 million (still quite high) and player 2's payoff from the tax is \$2 million.

There are four possible outcomes ranging from a tariff war to free trade. If neither player imposes a tariff, each can trade the full \$10 million worth of goods, and so each player would obtain a payoff of \$10 million. If player 1 imposes a tariff and player 2 does not, player 1 can export its \$10 million and obtain \$2 million in tariff revenue, so his payoff from this outcome is \$12 million. Player 2, on the other hand, loses \$2 million because of player 1's tariff and thus only gets \$6 million in revenue because he cannot sell all of his more expensive goods now but only \$8 million worth of them. Finally, if both players impose tariffs, each exports \$8 million worth of goods to the other, loses \$2 million in tariff payments, and gains \$2 million in tax income. That is, both get \$8 million.

To summarize, the utilities attached to the four outcomes are as follows (the units are all millions of dollars):

$$u_1(TF) = 12 \qquad u_1(FF) = 10 \qquad u_1(TT) = 8 \qquad u_1(FT) = 6$$
$$u_1(FT) = 12 \qquad u_1(FF) = 10 \qquad u_1(TT) = 8 \qquad u_1(TF) = 6$$

Recall that these utilities represent the preference orderings of the two players. Each player likes most keeping his own tariff while his partner removes his, followed by unfettered free trade, followed by a trade war. Each likes least the situation where he removes his tariff but his partner keep his.

Let's represent the situation with a table to help our analysis. In table 1, the rows represent player 1's actions and the columns represent player 2's actions. Each cell in the table is the result of the two players taking their corresponding actions. For example, the upper left cell is the outcome "free trade" because both players choose F. Similarly, the upper right cell is the outcome where player 1 (the row player) chooses F and player 2 (the column player) chooses T. That is, it represent the situation where player has no tariff but player 2 has.

As before, each player derives some utility from each outcome. Each outcome is therefore represented with a pair of numbers. The first number is player 1's utility from this

		Player 2 F	Player 2 T
Player 1 F	F	$10, 10$	$6, 12$
Player 1 T	T	$12, 6$	$8, 8$

Figure 1: The Tariff Game.

outcome, and the second number is player 2's utility from this outcome. For example, the upper left cell is the free trade, FF, outcome. Each player obtains a payoff of 10 from this outcome because $u_1(FF) = u_2(FF) = 10$, and so the pair is $(10, 10)$. Similarly, the upper right cell is the free/tariff, FT, outcome. Player 1 obtains 6 from this outcome because $u_1(FT) = 6$, and player 2 obtains 12 from this outcome because $u_2(FT) = 12$. The pair is therefore $(6, 12)$, which is what is listed in that cell in the table.

This table is called a "payoff matrix" because it lists the payoffs for both players. It is a compact and convenient way to specify the utilities.

Each player's action depends on what that player thinks the other player would do because the outcome is not determined by any one player but by the actions of the two.

Suppose you are player 1 and you are contemplating what the best course of action for you might be. That is, you want to figure out whether to choose F or T.

If you choose F, you will get 10 if player 2 chooses F and you will get 6 if player 2 chooses T. If you choose T, you will get 12 if player 2 chooses F and you will get 8 if player 2 chooses T. In each case, your payoff depends on player 2's choice. So, you have to form a **belief** about what player 2 is likely to do. That is, you now have to take into account the possible behavior of player 2.

- Suppose that for some reason you think that player 2 will choose F for sure. In that case, playing F gives you a payoff of 10 and playing T gives you a payoff of 12. You would rationally choose T.
- Suppose now that for some reason you think that player 2 will choose T for sure. In that case, playing F gives you a payoff of 6, and playing T gives you a payoff of 8. You would rationally choose T again.
- Suppose I thought that player 2 would not impose a tariff with probability p and would impose it with probability $1 - p$.

  The expected utility from action F equals the utility from outcome FF times the probability that player 2 chooses F plus the utility from outcome FT times the probability that player 2 chooses T. That is,

  $$EU_1(F) = 10p + 6(1 - p) = 4p + 6.$$

  The expected utility from action T equals the utility from outcome TF times the probability that player 2 chooses F plus the utility from outcome TT times the probability that player 2 chooses T. That is,

  $$EU_1(T) = 12p + 8(1 - p) = 4p + 8.$$

Note that $4p + 6 < 4p + 8$ regardless of the value of p because $6 < 8$ always. This means that

$$EU_1(F) < EU_1(T)$$

regardless of the value of p. That is, regardless of what belief player 1 has about player 2's action. Again, we find that player 1 is always better off choosing T instead of F no matter what player 2 does.

We conclude that player 1 would always choose T. If we go through the similar calculations for player 2 (and they are the same because the game is symmetric), we find that player 2 would always choose T as well.

The Tariff Game has a unique rational outcome which is TT, the trade war. This outcome is worse than the free trade outcome FF for both players. Yet it is the only rational solution to the situation!

This is a startling result. Both players would rather that FF was the outcome because they both would get \$10 million each. Still, acting fully rationally and in their self-interest, they arrive at the TT outcome where each only gets \$8 million instead.

2 The Consequences of Anarchy

The concept of anarchy has two components to it: no enforcement authority, and the possibility that force would be used. Let's see how the first one produces the result above.

We found a "solution" to the game where both players did what was in their best interest given that nobody could enforce an agreement. That is, the trade war outcome is self-enforcing because neither player wants to change his action given what the other player is doing.

Let's suppose that there was some international organization that could enforce agreements. That is, it could detect violations and punish them accordingly. Suppose that the two players write a contract in which they pledge to abolish all tariffs and maintain free trade. If anyone reneges from this contract and imposes a tariff, the organization will impose penalties worth \$3 million on the cheater. To put it simply, every time a player chooses T, it automatically pays \$3 million in penalties. The payoff matrix now becomes

		Player 2	
		F	T
Player 1	F	$10, 10$	$6, 9$
	T	$9, 6$	$5, 5$

Figure 2: The Tariff Game with Enforcement.

What is the optimal behavior now? Consider player 1's actions. Suppose player 2 chooses T: if player 1 plays T also, it will get 5, and if it plays F, it will get 6, and since $6 > 5$, the best response to the other player choosing T is to choose F. Suppose player 2 chooses F: if player 1 plays T it will get 9, and if it plays F, it will get 10, and since $10 > 9$, the best response to the other player choosing F is to choose F as well. Similar logic shows

that player 2 also strictly prefers to play F. Therefore, FF is the outcome of this game because both players are choosing their optimal behavior and neither has an incentive to change its action given what the other player is doing.

If there existed an international organization with the power to impose penalties like I just described, then free trade becomes possible. However, under conditions of anarchy such a cooperative arrangement cannot be made.

3 Equilibrium: Mutually Best Responses

Finding a solution to a game theoretic model boils down to finding **optimal behavior** for each participating actor. Optimal behavior means that the actor does its best given what the other player is doing. We find the actor's **best response** to the actions of the other player. A best response to some action is an action that yields the responding player the highest possible utility.

To find a player's best responses, we go through each possible action of the opponent and find the action that gives the player the highest utility. Of course, which of these best responses the player should choose depends on what it believes its opponent is going to do. Since its opponent is rational, it will do its best also. So, we must find the best responses of the opponent to each of the player's own actions.

Once we have the best responses we look for actions that are mutually best responses. That is, actions that are best responses to each other. The two mutual best responses are an **equilibrium** of the game. These actions form an equilibrium because they are such that given what one player is doing the other player has no incentive to change his action because he cannot do any better (he is already best-responding). They are sort of a "resting place" from which neither can move by itself to some other place that gives it a better payoff.

The equilibrium is a natural and intuitive solution to the game between two self-interested actors. The actions that they take in equilibrium, the **equilibrium actions**, produce an outcome called the **equilibrium outcome**. This is the outcome that we, as analysts, would use as our prediction for how the interaction is going to unfold. Finding equilibria for progressively more interesting models allow us to analyze optimal behavior in various settings and, as we saw above, find solutions that help overcome problems with incentives.

In the tariff game, the equilibrium action for each player is T and the equilibrium outcome is TT. The equilibrium is actually called **Nash equilibrium** in honor of John Nash who invented it.

4 Summary

- to analyze the strategic interaction of rational self-interested actors, we construct a **formal game-theoretic model**
- the model includes a description of the actors (preferences and beliefs) and the environment (actions and information)

- we solve the model by finding its **equilibria**
- an equilibrium is an action profile such that each player's action is a best response to all other players' actions
- in equilibrium no player has an incentive to unilaterally deviate by changing his action given what the other players are doing because he cannot improve his payoff by deviating
- finding the game's equilibria (solving the game) allows us to make predictions about the expected outcome of the strategic interaction
- equilibrium analysis also shows the reasons actors do what they do when they are doing their best in a strategic setting
- such analysis also shows how we might be able to change the incentives to produce behavior that we might be interested in (like cooperation in the tariff game)