Social Choice and Spatial Models of Policy

Many analysts take the state as the unit of analysis when it comes to important international events. So we talk about a Second Persian Gulf War between America and Iraq, or a crisis between the U.S. and North Korea, or bargaining for more money between Turkey and the U.S. In other words, we often take the state to be the important actor whose behavior we want to explain. It is in this context that you frequently hear the much abused and maligned term "the national interest." But what is it?

There are several possible ways we can approach the problem, and all of them have been used in international relations theory:

- Objective interest, which overrides all other concerns whether states realize that or not. For example, both realism and neorealism postulate that state survival is the most important national interest and all other goals are subordinate to this one. Liberals tend to argue that the world is not such a dreadful place and that economic well-being is the most important national interest.

- Expression of elite choice. In this view, elites have specific interests that they pursue through the state apparatus, to which they have better access than ordinary people. Elites then "sell" these policies to the rest of us, inducing our choices to conform to their preferences. This works both for democracies and non-democracies (authoritarian or totalitarian regimes).

- Expression of people's choice. Proponents of democracy argue that the national interest is simply an aggregation of individual preferences. That is, each and every one of us has his or her own preferences. In a democracy, we would then use some aggregation mechanism, usually voting, to arrive at the social preference.

Of course, there is no such entity as a state when it comes to preferences. States do not have preferences, people do. Instead of postulating an objective to an abstract entity (the state), we take the national interest to be really an expression of individual preferences, whether they are elite decision-making groups or voters. A state implements the "best" policy consistent with either elite or voter preferences. How do these groups decide what alternative is best?

1 Group Decisions: Preference Aggregation

Both voters and members of the elite are individuals who have individual preference orderings. We shall continue to assume that individual preference orderings obey the rationality rules of completeness and transitivity. When these groups must make decisions, they have to find ways of aggregating these individual preferences into a group preference and then decide on an action consistent with maximizing the utility given this group preference.

How do groups make decisions? Should groups depend on unanimity (consensus)? This would be best, of course, but people have different preferences and most often disagree about great many things. If we take consensus to be the necessary condition for a decision, groups will seldom do anything.

We can relax the unanimity requirement and agree that we only need some fraction of the group members to concur for the decision to be made by the group. We can set a certain threshold that must be crossed for a decision to be made legitimately by the group. A super-majority requirement would be something like needing 80% of the members to agree.

We are all familiar with the simple majority rule where an alternative that garners 50% of the votes plus 1 wins. But what about deciding among more than two alternatives? In these cases we usually require some sort of plurality. That is, we pick the alternative that receives the largest number of votes, even though it may fall short of a simple majority.

There are other, more complicated rules, that take into account the specific composition of the group. For example, the U.N. Security Council has 5 permanent members and 10 elected members who serve on a rotational basis. Each of the 5 permanent members has the power to veto any decision of the Council. However, for a decision to be made, the five votes are not sufficient because the rules require that at least nine of the current members of the Council vote for it. That is, for a substantive decision to be made in the SC, at least nine of the members must vote for it, and none of the five permanent members must vote against it (they can abstain).

Each member of the group has an **individual** preference ordering, which we assume to be rational. The group then uses an **aggregation rule** to take all individual preferences into account and create a **social** (or group) preference ordering. If this social ordering is rational (that is, it obeys the same minimal requirements as the individual preferences do), then it is well-defined and the group can take an action that maximizes that social preference.

1.1 An Example with Three Voters

Let me give you a very simple example to illustrate these things. Suppose the group consists of three people who have to decide among three alternatives. We can compactly represent the situation with a table that lists each individual and his ranking of the three alternatives:

		Person 1	Person 2	Person 3
Preference	*Best*	x	z	y
	So-so	y	x	z
	Worst	z	y	x

Let's use majority rule in paired comparisons as our aggregation method. The group votes on every pair of alternatives, selects a winner using majority rule, and then constructs the social ordering. Let's apply this to our example:

$$x, y \qquad\qquad x, z \qquad\qquad y, z$$

$$\left.\begin{array}{ll} 1: & x \succ y \\ 2: & x \succ y \\ 3: & y \succ x \end{array}\right\} \; x \succ y \qquad \left.\begin{array}{ll} 1: & x \succ z \\ 2: & z \succ x \\ 3: & z \succ x \end{array}\right\} \; z \succ x \qquad \left.\begin{array}{ll} 1: & y \succ z \\ 2: & z \succ y \\ 3: & y \succ z \end{array}\right\} \; y \succ z$$

The majority rule in binary comparisons produced the following social preference ordering: $x \succ y \succ z \succ x$. That is, the aggregation rule produced an irrational (because intransitive) preference ordering, or, as we sometimes call this phenomenon, *cycling*.

Why is this a problem? Because an actor with an irrational preference ordering can do anything and whatever he does will be consistent with maximizing his utility!

1.2 Agenda Setting

To make this clear and to illustrate an extremely troubling corollary to this result, suppose an agenda setter is selected to decide the order in which alternatives come up for vote. The agenda setter picks two alternatives and each member votes for its most preferred alternative of the two. The alternative that receives the majority votes wins and the losing alternative is discarded. The winning alternative is then pitted against the third remaining alternative and each member votes for the one it likes best between these two. After the second round, the alternative with the most votes wins.

Suppose we pick player 1 to be the agenda setter. His most preferred alternative is x and he can ensure that the group selects it! He constructs the agenda so that the group first votes on y and z. Since $y \succ z$, the winning alternative is y. The group then votes on y and x, and since $x \succ y$, x becomes the winner.

Suppose, however, that we picked player 2 to be the agenda setter. His most preferred alternative is z, so he selects x and y, which leaves x as the winner of the first vote. In the second vote z beats x, and so the group selects z as the winner.

We are not done yet. Suppose we picked player 3 to be the agenda setter. His most preferred alternative is y, so he selects x and z, which leaves z as the winner of the first vote. In the second vote z loses against y, and the group selects y as the winner.

Thus, depending on *identity of the agenda-setter*, the group can arrive at *any of the possible alternatives* as its choice! Very troubling indeed.

2 Arrow's Impossibility Theorem

You might be tempted to discard this example as irrelevant or rare. In particular, you might wonder whether this was not an artifact of the extreme differences in player individual preferences. Or you might wonder whether another aggregation rule could have been used to guarantee transitivity of the social preference.

You will be right if you thought that the problem has to do with the extreme differences among the group members. However, there is a general result that demonstrates that without restricting these preferences, there exists *no aggregation rule that will guarantee a rational social preference ordering unless the group simply selects a dictator and implements his rational preference ordering* (recall that all individual preferences are rational). This result is due to economist Kenneth Arrow, who won the Nobel prize.

ARROW'S IMPOSSIBILITY THEOREM. *There exists no mechanism for aggregating rational individual preferences into a rational social preference ordering that satisfies the following four conditions:*

Universal Domain *No individual preference ordering is excluded. Every logically possible combination of individual orderings is allowed.*

Pareto Optimality *If at least one member prefers x to y and everyone else either prefers x to y also or is indifferent between them, then the group preference must reflect a preference for x over y as well.*

Independence of Irrelevant Alternatives *All that is relevant for the social ordering of any two alternatives x and y are the individual orderings of x and y, and is independent of the individual orderings of x and z, for example.*

Non-dictatorship *There is no individual whose preferences dictate the group's preferences independent of the other members.*

Any group choice mechanism (aggregation rule) that satisfies Universal Domain, Pareto Efficiency, and IIA is either dictatorial or does not guarantee rationality. The theorem does *not* say that the social preference ordering will *always* be irrational, just that we cannot guarantee group rationality in all situations.

What does this imply for our study of international relations?

- Assuming rational actors at any level of aggregation is extremely problematic. Not only states, but bureaucracies, international organizations, sub-state associations, every possible group you can think of may be subject to irrationality.

- Even if we assume that states are rational actors (e.g. a monarchy or another type of absolutist regime, like dictatorship), groups of states still face social choice problems (U.N., NATO, IMF).

- National interest (especially in a democracy) cannot be defined because states usually have a large number of individuals with heterogeneous preferences and they have to deal with a large number of options. All of these combine to make group irrationality very likely.

3 Black's Median Voter Theorem

So far we have discussed only discrete alternatives but often it is more natural to think about alternatives as placed along a continuum. For example, the annual level of spending on the military can vary from, say, 0 to several billion. We need a convenient way of describing an individual's utility associated with different levels of military spending.

We place the amount of military spending along a line. The individual will have at least one level of spending he likes most, which we shall call his **ideal point**. This is the level of spending that yields this actor his highest utility. As the outcome moves away from this ideal point, the utility uniformly declines. A simple example is shown in Figure 1.

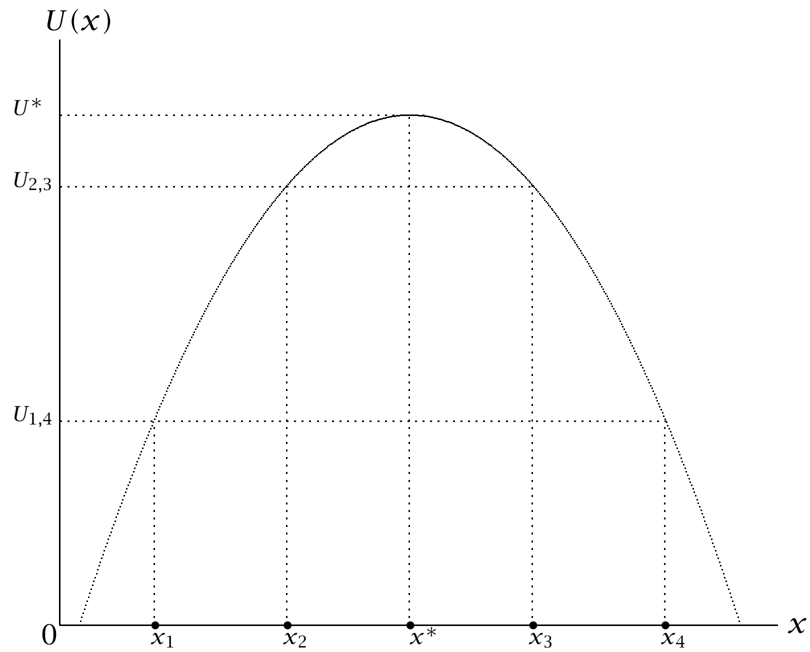

Figure 1: Preferences in Single Dimension. Level of military spending denoted by x.

The level of military spending denoted by x^* is the individual's ideal point because it yields the highest utility. Any other level of spending is associated with a smaller utility and the further the level from x^*, the smaller the utility, regardless of whether the levels are increasing or decreasing. For example, at x_2 the utility is $U_{2,3}$, which is strictly worse than U^*, and at x_1, it is $U_{1,4}$, which is even worse. Generally speaking, the further the outcome from the individual's ideal point, the worse it is.

Preferences of this type are called **single-peaked** because they reach one high point on the graph and fall continuously from there. This means that single-peaked preferences can be represented by a straight line (that either rises or falls), or by a curve that first rises and then falls without rising again. So, preference of the type in Figure 2 would be ruled out.

Assuming single-peaked preferences for all individuals *violates the Universal Domain requirement* because we are excluding certain types of preferences. Not surprisingly, once we violate one of the crucial necessary conditions of Arrow's Theorem, its stark conclusion no longer holds. The result is due to several people, but the most famous formulation is the one by Duncan Black:

Median Voter Theorem. *If individual preferences are single-peaked on a single*

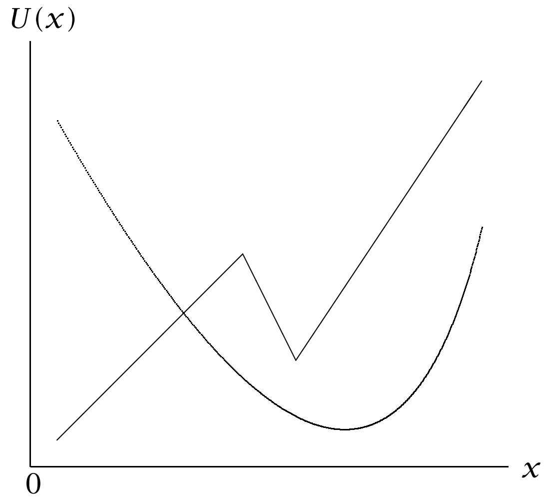

Figure 2: Non Single-Peaked Preferences.

dimension and there is an odd number of voters, then the social preference ordering under majority rule is transitive, and the median ideal point is the winner.

To see how this works, let's look at an example with just three voters whose single-peaked preferences are shown in Figure 3. Arranging the preferences along this single dimension (military spending), yields a sequence of ideal points (denoted by x_1^*, x_2^*, and x_3^*), with x_2^* being the ideal point of the **median voter**. The voter is called "median" because there is an equal number of voters with ideal points on each side of his ideal point.

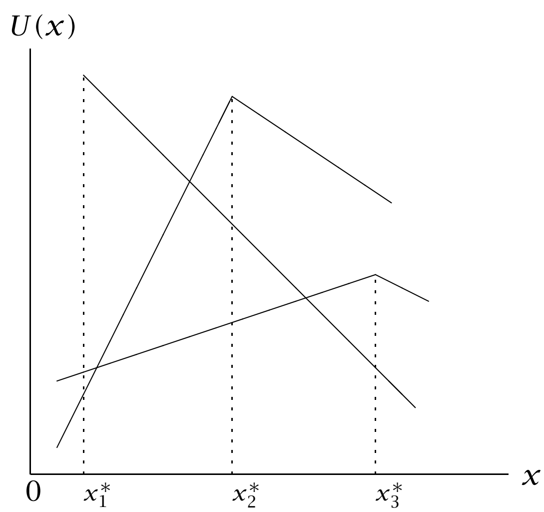

Figure 3: The Median Voter Theorem.

The theorem tells us that the social preference ordering will be rational and that x_2^* will be the winner of any voting. To see that this is the case, suppose the group has to vote on alternatives x_1^* and x_2^*. Player 1 will vote for x_1^* but both other players will vote for x_2^*, and it will win. Therefore, $x_2^* \succ x_1^*$.

Suppose now that they have to vote on x_2^* and x_3^*, in which case player 3 votes for x_3^* but both other players vote for x_2^*, which wins again. Therefore, $x_2^* \succ x_3^*$.

Suppose now that they have to vote on x_1^* and x_3^*, in which case players 2 and 3 vote for x_3^* which beats x_1^* which only gets player 1's vote. Therefore, $x_3^* \succ x_1^*$.

This yields the social preference ordering $x_2^* \succ x_3^* \succ x_1^*$, which is transitive and complete, and therefore rational. This is always going to be the case regardless of how the vote is conducted. We are not restricted to voting on ideal points, of course, but you should satisfy yourself that the result holds for arbitrary points along the x axis. It is important to realize that x_2^* will beat *every other alternative* under majority rule when paired with it.

Intuitively, for any alternative $x < x_2^*$ both players 2 and 3 would vote for x_2^* and against x, while for any $x' > x_2^*$, both players 1 and 2 would vote for x_2^* and against x'. Once x_2^* is brought to a vote, it will always be the outcome.

Why is this important? Because it tells us that if preferences are single-peaked, social preferences are rational, and social choice is well-defined. Even more, it tells us that the median voter is decisive in the sense that the expected outcome will reflect his ideal point. Thus, all one would need to know in order to analyze group decisions under majority rule (assuming single peakedness) is the position of the median voter within that group.

4 McKelvey's Chaos Theorem

The Median Voter Theorem holds for single dimensions. But most policies are not about a single issue. Rather, they are packages that deal with multiple issues simultaneously. For example, a government spending policy would include welfare spending in addition to the military one.

It is relatively easy to define preferences over multiple dimensions using ideas from the one-dimensional case. It is possible to define a version of single-peakedness as well but it does not help. Richard McKelvey proved that a winning alternative will rarely exist when multi-dimensional policies are concerned.

> THE CHAOS THEOREM. *In multi-dimensional policy spaces, using paired comparisons and majority rule, winning alternatives will rarely exist, and if they do not exist, any policy can be chosen with the appropriate agenda.*

McKelvey's theorem asserts that (a) when there are more than one dimensions to a policy, the social preference ordering is likely to be intransitive, and (b) by manipulating the agenda, the polity can choose anything! That is, group choice becomes completely unpredictable again and, what's perhaps worse, subject to strategic manipulation by a smart agenda-setter.

Because individually rational preferences can lead to irrational social preferences, strategy can be used to control the group's decisions by manipulating the agenda. That is, a member of the group is powerful if he can get the group to implement his particular preferences. To understand state preferences, we must look at individual preferences, institutions that determine voting rules, and the power of players within these institutions.

What do we conclude from all this? Group choice may be subject to a host of problems, the worst of which is the cycling problem caused by intransitivity of the social preference ordering. If policies can be reduced to a single dimension and if individual preferences are single-peaked along this dimension, then we can predict group choice, which will be the median voter's ideal point. If neither of these requirements hold, the social preference ordering will be irrational and the polity can choose just about anything. It also becomes subject to strategic manipulation of agenda-setters.

It is interesting that the world does not exhibit wild cycling and instability as predicted by the chaos theorem. Studies have shown that in many cases policies can, in fact, be reduced to a single dimension while retaining single-peakedness. People can also design institutions to help overcome some of the instability by restricting either the number of alternatives under consideration or the admissible preference orderings. (The committee system, by the way, is one such structural method of restricting these.) In addition, relatively homogenous groups — that is, groups whose members do not differ too much — may be less prone to cycling.

5 Summary

- To define important concepts such as the **national interest**, we must understand how groups make decisions.
- All members of a group have **rational individual preference orderings**. The group uses a **preference aggregation rule**, like majority vote, to construct the **social preference ordering**. The alternative that is most preferred by the group wins.
- ARROW'S IMPOSSIBILITY THEOREM shows that if the social ordering satisfies Universal Domain, Pareto Optimality, and Independence of Irrelevant Alternatives, then there exists no non-dictatorial aggregation rule that can guarantee a rational social preference ordering.
- We can represent preferences with utility functions. An individual preference is **single-peaked** if there exists one point that yields the highest utility, the individual's **ideal point**, and if utility uniformly diminishes the further the alternatives get from this point.
- BLACK'S MEDIAN VOTER THEOREM shows that if individual preferences are single-peaked along one dimension, then the social preference ordering under majority rule is rational. The winning alternative is the ideal point of the **median voter**.
- MCKELVEY'S CHAOS THEOREM shows that in multi-dimensional settings under majority rule social preference orderings will generally be intransitive, in which case any policy can be chosen by using the appropriate **agenda**.
- Empirically it appears that many policies can be reduced to a single dimension, in which case the Median Voter Theorem provides a useful approximation of the policy chosen.