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Cheap Talk Can Matter in Bargaining*

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This paper describes an intuitive way in which cheap talk can matter in a two-stage bargaining game in which talk may be followed by serious negotiation. The intuition that all buyers would claim to have low reservation prices is incorrect in our model. Instead, if good-faith participation is endogenously determined then the parties can use talk to trade off bargaining position against the probability of continued negotiation. Our cheap-talk equilibrium features bargaining behavior that could not be equilibrium behavior in the absence of talk. *Journal of Economic Literature* Classification Numbers: 026, 022.

1. INTRODUCTION

Since the seminal work of Spence [17], economists have understood how an informed agent's choices may reveal private information if they affect him differently depending on what he knows. This idea of costly signaling has been extremely influential in recent economic theory, underlying analyses of everything from education to entry deterrence. But perhaps its greatest influence has been in the noncooperative theory of bargaining: a large and growing literature, beginning with Fudenberg and Tirole [7] and Sobel and Takahashi [16], analyzes how bargainers can improve their

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terms of trade by undertaking costly actions (notably delay) meant to convince their "opponent" that their interest in trade is at best lukewarm.

While such communication through costly signals is undoubtedly important, much communication also occurs through costless words, or *cheap talk*. Talk is ubiquitous and is often listened to, even where no real penalty attaches to lying, and where claims do not directly affects payoffs. Crawford and Sobel [2] showed formally that such cheap talk can be credible in equilibrium if the parties have at least some interests in common.

Despite Crawford and Sobel's result, one might expect that cheap talk could not matter in bargaining, because each side has an obvious incentive to seem unenthusiastic: we think of a bazaar, where the buyer sneers at the seller's goods, which the latter declares are so precious as to be scarcely for sale. Yet casual observation reveals much cheap talk of the opposite kind: people often claim to be "seriously interested" in trading. Unless this is mere meaningless noise, something is missing in the theory.

One possibility is that these claims—or confessions—of urgent desire to trade are meant to encourage the other side to participate in more detailed negotiation. As we show below, if saying that one is "keen" makes one's partner more likely to negotiate, then it is the keenest types (high-value buyers, low-value sellers) who are most willing to say so, damaging as it is to their terms of trade if trade occurs.

The following story illustrates the element of common interest that drives our analysis: cheap talk can affect whether negotiation ensues. Imagine that one Saturday evening, two corporate moguls have a chance encounter at their country club. One mogul's company owns a division that the other mogul's firm may wish to buy. Serious negotiation, involving binding offers and hordes of lawyers, can take place on Monday morning; all that can happen Saturday night is talk. If, based on this talk, the moguls conclude that there is sufficient prospect of gains from trade, then they will send their lawyers into the fray on Monday morning. Otherwise, Saturday evening will be the end of it. Therefore, each mogul has an incentive not to sneer too much, lest the other choose not to try to do business with one who seems uninterested. The strategy (common in bazaars) of sneering and then returning for serious bargaining is less attractive to the moguls because a sneer may end negotiations.

In this paper, we turn this basic intuition into a precise equilibrium statement in a particular bargaining model, the sealed-bid double auction studied by Chatterjee and Samuelson [1]. We analyze a two-stage game: talk comes first, then formal (binding) negotiation. Not only is information conveyed by cheap talk in equilibrium, but the equilibrium mapping from the buyer's and seller's reservation values to outcomes (whether trade occurs and if so at what price) differs from any that could occur in an equilibrium without talk. Further, some outcomes of the "talk" stage lead to second-stage bidding strategies that could not be equilibrium strategies absent the changes in beliefs that the talk causes.

To complete this Introduction, we describe the differences between our cheap-talk equilibrium and the literatures on bargaining and on mechanism design. As described above, in most game-theoretic analyses of bargaining, communication takes place only through actions that can directly affect payoffs, and that therefore can be costly signals. Typically, such an action either directly imposes costs of delay, or directly affects payoffs by constituting an offer that is binding if the other player accepts it, or both. Cheap talk does neither of these things. Of course, *in equilibrium*, different types have incentives to choose different cheap-talk messages, but no part of these incentives consists of *exogenous* costs or benefits. This distinguishes cheap talk from signaling, and distinguishes our analysis from a standard bargaining game.

Cheap talk also differs from a mechanism (in the sense of Myerson and Satterthwaite [13]). In a mechanism, messages without direct costs are used, but a mediator controls the communication and is committed to enforcing a given outcome as a function of the messages. In cheap-talk equilibrium, by contrast, no agent can commit to a choice of outcome as a function of messages; rather, the outcome must be a perfect Bayesian equilibrium given the information conveyed by the messages. Moreover, these messages become common knowledge, whereas a Myerson–Satterthwaite mediator can, and typically does, limit the information he passes on to the players. Of course, every cheap-talk equilibrium can be implemented as a mechanism, but in general the converse is not true. Matthews and Postlewaite [11] explore the extent to which this converse holds in a double auction. We discuss their work in more detail in Section 2.

2. A Cheap-Talk Equilibrium

This section analyzes cheap talk in a well-known model of bargaining under incomplete information. In terms of our story of the two corporate moguls, if the parties do meet on Monday, they play the following sealed-bid double auction. Buyer and seller name prices p_b and p_s, respectively, and trade takes place at price $(p_b + p_s)/2$ if $p_b \geqslant p_s$; otherwise, there is no trade.[1]

[1] For those who miss the lawyers, consider the commitment necessary to play even this simple game: what, for instance, stops one party from reneging on his offer in order to capitalize on the information conveyed by the other party's offer?

Before the double-auction stage of our two-stage game there is a cheap-talk stage. (On Saturday the moguls can engage in cheap talk.) We consider the simplest possible language: each party can claim either to be "keen" or to be "not keen." We assume that these claims are made simultaneously. We emphasize that these claims do not directly affect payoffs: they work *only* by affecting the other player's beliefs. In particular, they are not commitments nor are they verifiable.

To summarize, the extensive form is as follows. First, the parties simultaneously announce whether they are "keen" or "not keen"; these announcements do not directly affect either party's payoff. After observing the pair of announcements, the parties play the double auction described above. If trade takes place at price p, then a buyer with valuation v_b achieves payoff $v_b - p$ and a seller with valuation v_s achieves payoff $p - v_s$; if trade does not occur then payoffs are zero.

The extensive form of our formal model differs slightly from our informal story about the corporate moguls: in the formal model, the double auction is played after any pair of cheap-talk announcements, whereas in the informal story, Saturday evening could be the end of it. Intuitively, we think that a player who is made sufficiently pessimistic about the likely gains from trade will not bother to participate in bargaining. This is, of course, because there are *costs* to such participation: both disbursements (on lawyers, etc.) and opportunity costs (notably alternative negotiations foregone). We do not model these costs, since doing so would complicate the model and obscure the basic tradeoff. Even so, we could simply posit that insufficiently encouraged traders do not show up to bargain on Monday: although (absent costs) this choice is weakly dominated, it is still an equilibrium for neither side to show up (since trade cannot happen unless both sides appear). Alternatively, we can think of the parties playing the following "no-trade" equilibrium in the second stage after a discouraging conversation: the buyer bids $p_b = 0$, and the seller bids $p_s = 1$. Whichever choice the reader prefers, there is no trade in such a subgame.[2]

Chatterjee and Samuelson [1] analyze a class of equilibria of the double auction without cheap talk. They show that bounded, strictly monotone, and differentiable equilibrium strategies must satisfy a linked pair of differential equations. In the standard case in which v_s and v_b are independently and uniformly distributed on [0, 1], these differential equations have a solution in which both the buyer and the seller play linear strategies. We

[2] A third rationale for there being no trade after both players say "not keen" is that they need to coordinate on when and where to meet on Monday, and an attempt to arrange a meeting belies a party's claim that he is "not keen." Note that this is *not* the same as saying that talk determines whether the parties *can* meet; such talk would not be cheap talk. Here the set of times and places available for a meeting is independent of the talk, but is large enough and sufficiently lacking in focal points that meeting without agreement is unlikely.

call this the Chatterjee–Samuelson equilibrium and we use it to define equilibrium behavior whenever possible. There are, however, many other equilibria in a double auction in the absence of cheap talk (including the "no-trade" equilibrium above), only some of which satisfy the conditions that Chatterjee and Samuelson assumed. Leininger *et al.* [9] and Satterthwaite and Williams [14] explore some of these alternative equilibria, and we use one in our model when it is not possible to use the Chatterjee–Samuelson equilibrium.

In our game, as in every cheap-talk game, there is an uncommunicative (or "babbling") equilibrium: if cheap talk is taken to be meaningless, then parties are willing to randomize uninformatively over the possible messages. But there are also two more interesting equilibria in which cheap talk is meaningful. In one, serious bargaining takes place only if both parties claim to be "keen"; in the other, a single such claim suffices. In both of these equilibria, serious bargaining does not occur if neither party claims to be "keen".

In the first of these equilibria with meaningful cheap talk, the Chatterjee–Samuelson equilibrium reappears: everyone claims to be "keen" except those types who are sure not to trade.[3] In this equilibrium, cheap talk is credible, but does not affect the equilibrium outcome: the mapping from type-pairs to bids and probability of trade is the same as in the Chatterjee–Samuelson equilibrium, which has no cheap talk.

In the other equilibrium, however, cheap talk matters in an important way: low-value buyers and high-value sellers are willing to jeopardize continued negotiation so as to improve their bargaining position; those who have more at stake cannot afford this risk. We focus on this equilibrium because it involves second-stage bidding strategies that could not be equilibrium strategies in the absence of talk.

We analyze our equilibrium in the standard case in which v_s and v_b are independently and uniformly distributed on $[0, 1]$. We show in the Appendix that the following strategies for the cheap-talk stage are part of a perfect Bayesian equilibrium. Buyers above the critical type

$$y = \frac{22 + 12\sqrt{2}}{49} = 0.795$$

say "keen" while those below say "not keen." Sellers below $(1 - y)$ say "keen," while those above say "not keen."

If both parties say "not keen" then the negotiation effectively ends, as

[3] In the standard case in which v_b and v_s are independently and uniformly distributed on $[0, 1]$, all buyers with $v_b > \frac{1}{4}$ and all sellers with $v_s < \frac{3}{4}$ claim to be "keen." Strictly, the other types of buyers and sellers, who will not trade, may say anything. But if there are any costs of serious bargaining, then they must say "not keen."

discussed above. If at least one says "keen" then bargaining continues with a (possibly asymmetric) sealed-bid double auction. If, for instance, the seller says "not keen" and the buyer says "keen" then it becomes common knowledge that the buyer believes that the seller's type is above $1 - y$ and that the seller believes that the buyer's type is above y, and negotiation proceeds on that basis. Similarly, if the seller says "keen" and the buyer says "not keen" then it becomes common knowledge that the buyer believes that the seller's type is below $1 - y$ and that the seller believes that the buyer's type is below y. In both of these cases, we use the Chatterjee–Samuelson equilibrium to solve the resulting bargaining game. Finally, if the buyer and the seller both say "keen" then it becomes common knowledge that the buyer believes that the seller's type is below $1 - y$ and that the seller believes that the buyer's type is above y. In this case, the linear Chatterjee–Samuelson equilibrium breaks down because gains from trade are guaranteed ($y > \frac{5}{8}$). Because of this, and because the subgame is symmetric, we focus on the equilibrium in which trade occurs with certainty at a price of $\frac{1}{2}$. Formally, this is a "one-step" equilibrium (see Leininger *et al.* [9]): the buyer bids $\frac{1}{2}$ if his value is above $\frac{1}{2}$ but otherwise bids 0, and the seller follows an analogous strategy. In the Appendix we state and prove Lemma 1, which describes in more detail the cheap-talk and bargaining behavior in our cheap-talk equilibrium.

Propositions 1 and 2 show that cheap-talk really matters in our equilibrium. (Proofs of these results are given in the Appendix.) Proposition 1 shows that the equilibrium mapping from pairs of types (v_b, v_s) to outcomes (whether trade occurs, and if so at what price) differs from any that could occur in a no-talk equilibrium, because talk achieves some correlation of bids.

PROPOSITION 1. *The equilibrium mapping from types to outcomes in our cheap-talk equilibrium cannot arise in an equilibrium without talk.*

Proposition 2 shows that cheap talk matters in our equilibrium in a more fundamental way: in the {"not keen," "keen"} and {"keen," "not keen"} subgames, the bidding stategies we specify form an equilibrium only because talk changes each party's belief about the other's type.

PROPOSITION 2. *There does not exist an equilibrium without cheap talk in which sellers* $v_s \in [1 - y, 1]$ *and buyers* $v_b \in [y, 1]$ *name the prices that they name in the {"not keen," "keen"} subgame of our cheap-talk equilibrium.*

We close this section by discussing the related work of Matthews and Postlewaite, who also analyze cheap talk preceding a double auction. They characterize the equilibrium outcomes that can be obtained through both mediated communication (as in mechanism design) and unmediated communication (as in our definition of cheap talk). Their constructive proofs of these characterization results restrict attention to a simple form of unmediated-communication equilibrium: talk simply coordinates different types on different equilibria, each of which is an equilibrium without talk. This kind of coordination is impossible in an equilibrium without talk.

Proposition 1 summarizes the role of this Matthews–Postlewaite-style coordination in our cheap-talk equilibrium. Proposition 2 makes precise our claim that in our equilibrium cheap talk is credible because it allows players to trade off bargaining position against the probability of continued negotiation. Different types view this tradeoff differently, and credible talk results from these differences. This intuition is not brought out in the Matthews–Postlewaite analysis, although their characterization does include our equilibrium.

3. Welfare

This section compares the payoffs in our cheap-talk equilibrium to those in the Chatterjee–Samuelson equilibrium. Calculation shows that our cheap-talk equilibrium yields buyer-type v_b an interim payoff, evaluated before the cheap-talk phase, of

$$W_b(v_b) = \begin{cases} 0 & \text{if } v_b \leqslant \frac{1}{4}y, \\ \frac{1}{2}(v_b - \frac{1}{4})^2 & \text{if } \frac{1}{4}y < v_b \leqslant 1 - \frac{3}{4}y, \\ (1-y)(v_b - \frac{1}{2} + \frac{1}{4}y) & \text{if } 1 - \frac{3}{4}y < v_b \leqslant y, \\ \frac{1}{2}(v_b - \frac{1}{4}y)^2 - \frac{1}{2}(\frac{7}{4}y - 1)^2 & \text{if } v_b > y. \end{cases}$$

An immediate consequence is that if $\frac{1}{4}y = 0.199 < v_b \leqslant \frac{1}{4}$, then buyer-type v_b is strictly better off than in the Chatterjee–Samuelson equilibrium. In fact, many other types are better off in our equilibrium than in Chatterjee–Samuelson. Equating our $W_b(v_b)$ to the Chatterjee–Samuelson equivalent $W_b^{CS}(v_b) \equiv \frac{1}{2}(v_b - \frac{1}{2})^2$ yields a unique crossover point, which lies in the range $1 - \frac{3}{4}y < v_b < y$, given by the solution v_b to

$$(1-y)(v_b - \tfrac{1}{2} + \tfrac{1}{4}y) = \tfrac{1}{2}(v_b - \tfrac{1}{4})^2,$$

which is approximately equal to 0.599. Thus, all buyer types in (0.199, 0.599), and similarly all seller types in (0.401, 0.801), are better off with cheap talk. In fact, exactly as many types strictly prefer our equilibrium as strictly prefer Chatterjee–Samuelson. (Types who never trade are of course indifferent.)

The pairs (v_b, v_s) who trade in our equilibrium are illustrated in Fig. 1, which also shows the corresponding region for the Chatterjee–Samuelson equilibrium. The (ex-ante) probability that (v_b, v_s) falls in the trading region for our equilibrium is $\frac{3}{2}y(1-y)$, or approximately 0.244, somewhat

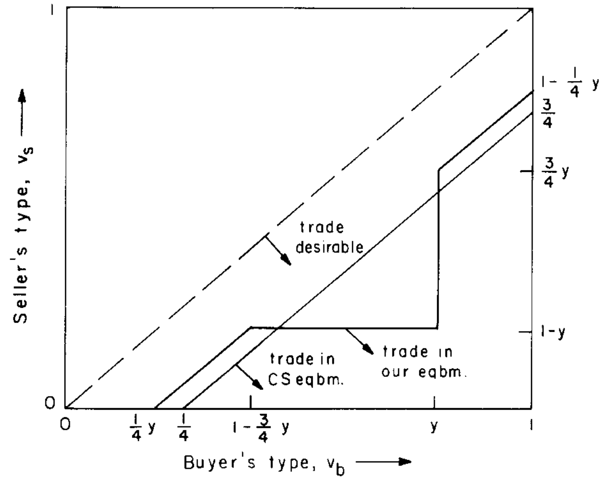

FIGURE 1

less than the corresponding probability (0.281) for the Chatterjee–Samuelson equilibrium: our equilibrium involves less trade. Similarly, the ex-ante expected total gains from trade in our equilibrium[4] are approximately 0.124, less than the Chatterjee–Samuelson figure of 0.140.

Both of these comparisons are special cases of Myerson and Satterthwaite's general result that the Chatterjee–Samuelson equilibrium maximizes both the ex-ante probability of trade and the ex-ante gains from trade (for the independent, uniform case we have analyzed). Myerson [12], however, convincingly argues that such ex-ante efficiency is often irrelevant, because there is seldom an opportunity to make binding arrangements ex-ante (i.e., before either player knows his "type"). Myerson gives an example of an incentive-compatible mechanism (for the independent, uniform case) in which even high-value sellers (and low-value buyers) trade with positive probability, and therefore are better off than in the Chatterjee–Samuelson equilibrium. Our cheap-talk equilibrium is in the same spirit, but is derived from an extensive-form game in which talk takes place without a mediator.[5]

[4] The ex-ante expected total gains from trade in our equilibrium are given by

$$(1-y)^2 \cdot \frac{8-11y}{12} + \left(1-\frac{1}{4}y\right)^2 \cdot \frac{2+y}{12} - \frac{1}{3}\left(\frac{3}{4}y\right)^3 + \frac{1}{2}(1-y)\left[y^2 - \left(1-\frac{3}{4}y\right)^2\right].$$

[5] Of course, the subsequent sealed-bid double auction does require a mediator, both to accept simultaneous reports and to force the parties to walk away if trade is not prescribed (even if trade would be efficient). But such a game is hardly central to the tradeoff between bargaining position and the probability of continued negotiation that allows cheap talk to be credible in our analysis.

4. Conclusion

We believe that the economic importance of costless, nonverifiable, informal communication is much greater than its role in the literature suggests. The seminal work by Crawford and Sobel is justly famous, but applications have only recently begun to appear.

This paper introduces cheap talk to bargaining games. We emphasize that cheap talk can matter in bargaining when participation is endogenous, because the parties can then use talk to trade off bargaining position against the probability of continued negotiation. This talk can matter in an essential way: the cheap-talk equilibrium we analyze features bargaining outcomes that could not be equilibrium behavior in the absence of talk. Not surprisingly, this intuition does not require anything as specific as a sealed-bid double auction. See Farrell and Gibbons [4] for a more general discussion of these issues.

Cheap talk is important in many other economic settings as well. Farrell [3], for instance, studies cheap talk between potential entrants in a natural monopoly, Farrell and Saloner [5] consider cheap talk between potential adopters of a new technology, Forges [6] analyzes cheap talk in a hiring and job-assignment game, Gibbons [8] models interest arbitration as a cheap-talk game, Matthews [10] describes presidential rhetoric as a cheap-talk veto threat in the budgetary process, and Sobel [15] develops a theory of credibility in finitely repeated relationships. The fundamental insight that cheap talk can be credible in variable-sum games, combined with the ubiquity of such talk, suggests that a rich collection of other applications lies ahead.

Appendix

In this Appendix we adapt the Chatterjee–Samuelson analysis to suit our purposes, and then use the results to derive the equilibrium value of y. We give a formal statement (and proof) of the equilibrium cheap talk and bargaining behavior in Lemma 1. We also prove Propositions 1 and 2.

Consider a sealed-bid double auction in which it is common knowledge that the seller-type v_s is uniformly distributed on $[\underline{v}_s, \bar{v}_s]$ and buyer-type v_b is independently and uniformly distributed on $[\underline{v}_b, \bar{v}_b]$. Both parties name prices, p_s and p_b, and trade occurs at the average of the two prices if the buyer's price exceeds the seller's.

As Chatterjee and Samuelson show, an essential part of a bounded, monotone, and differentiable equilibrium is the solution of a linked pair of differential equations, and one solution (on which we and they focus) is linear:

$$\tilde{p}_s(v_s) = \tfrac{2}{3}v_s + \tfrac{1}{4}\bar{v}_b + \tfrac{1}{12}\underline{v}_s,$$
$$\tilde{p}_b(v_b) = \tfrac{2}{3}v_b + \tfrac{1}{4}\underline{v}_s + \tfrac{1}{12}\bar{v}_b. \tag{1}$$

Three cases must be considered. First, if the use of these strategies would imply that no type of either party is sure to trade (i.e., $\tilde{p}_b(\bar{v}_b) \leqslant \tilde{p}_s(\bar{v}_s)$ and $\tilde{p}_s(\underline{v}_s) \geqslant \tilde{p}_b(\underline{v}_b)$), then the equilibrium strategies are $p_s(v_s) = \tilde{p}_s(v_s)$ and $p_b(v_b) = \tilde{p}_b(v_b)$. Second, if the use of these strategies would make some types of at least one party sure to trade, but would not make all types of both parties sure to trade, then the buyer-type v_b names the price $p_b(v_b) = \min(\tilde{p}_b(v_b), \tilde{p}_s(\bar{v}_s))$ and the seller-type v_s names the price $p_s(v_s) = \max(\tilde{p}_s(v_s), \tilde{p}_b(\underline{v}_b))$. Third, if the use of these strategies would make all types of both players sure to trade, then the Chatterjee–Samuelson equilibrium breaks down. A continuum of equilibria still exist, but these equilibria are unrelated to the bidding strategies described in (1). In some of these equilibria, all types of both parties name a given price in the interval $[\bar{v}_s, \underline{v}_b]$ and trade occurs with certainty. We deal with this case below.

When no seller type is sure to trade, calculation shows that the buyer's interim payoff is

$$U_b(v_b; [\underline{v}_s, \bar{v}_s], [\underline{v}_b, \bar{v}_b]) = \begin{cases} 0 & \text{if } v_b \leqslant \underline{\beta}, \\ \dfrac{(v_b - \underline{\beta})^2}{2(\bar{v}_s - \underline{v}_s)} & \text{if } \underline{\beta} < v_b \leqslant \bar{\beta}, \\ v_b - \beta + \dfrac{\bar{v}_s - \underline{v}_s}{2} & \text{if } v_b > \bar{\beta}, \end{cases} \tag{2}$$

where $\Delta = (\bar{v}_b - \underline{v}_s)/4$, $\underline{\beta} = \underline{v}_s + \Delta$, and $\bar{\beta} = \bar{v}_s + \Delta$. (In this notation, no seller type is sure to trade when $\underline{v}_b < \underline{\beta}$.) The three cases in (2) correspond to the cases in which the buyer, given v_b and the supports of the players' types, is sure not to trade, might trade, or is sure to trade, respectively.

When some but not all seller types are sure to trade (i.e., $\underline{\beta} < \underline{v}_b < \bar{\beta}$), an interval of seller types trade with the lowest buyer type. The interim payoff to $\underline{v}_b$ is then

$$\underline{U}_b \equiv U_b(\underline{v}_b; [\underline{v}_s, \bar{v}_s], [\underline{v}_b, \bar{v}_b]) = \frac{(\underline{v}_b - \underline{v}_s - \Delta)^2}{3(\bar{v}_s - \underline{v}_s)}, \tag{3}$$

and the interim payoff for other buyer types is

$$U_{\mathrm{b}}(v_{\mathrm{b}};[\underline{v}_{\mathrm{s}},\bar{v}_{\mathrm{s}}],[\underline{v}_{\mathrm{b}},\bar{v}_{\mathrm{b}}])=\begin{cases}\dfrac{(v_{\mathrm{b}}-\underline{v}_{\mathrm{s}}-\Delta)^2}{2(\bar{v}_{\mathrm{s}}-\underline{v}_{\mathrm{s}})}-\frac{1}{2}\underline{U}_{\mathrm{b}} & \text{if } v_{\mathrm{b}}\leqslant\bar{\beta},\\ v_{\mathrm{b}}-\bar{\beta}-\frac{1}{2}\underline{U}_{\mathrm{b}}+\dfrac{\bar{v}_{\mathrm{s}}-\underline{v}_{\mathrm{s}}}{2} & \text{if } v_{\mathrm{b}}>\bar{\beta}.\end{cases} \tag{4}$$

Finally, when all seller types are sure to trade ($\bar{\beta}<\underline{v}_{\mathrm{b}}$), then all buyer types also are sure to trade, and the Chatterjee–Samuelson equilibrium breaks dows: the bids given by (1) are irrelevant, and the strategies $p_{\mathrm{b}}(v_{\mathrm{b}})=\tilde{p}_{\mathrm{s}}(\bar{v}_{\mathrm{s}})$ and $p_{\mathrm{s}}(v_{\mathrm{s}})=\tilde{p}_{\mathrm{b}}(\underline{v}_{\mathrm{b}})$ are *not* an equilibrium. Without proposing a general theory for this problem, we note that the subgame $\{v_{\mathrm{s}}\in[0,1-y],\ v_{\mathrm{b}}\in[y,1]\}$ is symmetric about $\frac{1}{2}$ and so (when $y>\frac{1}{2}$) it is natural to assume that trade will occur with certainty at a price of $\frac{1}{2}$. Then a buyer-type $v_{\mathrm{b}}\geqslant\frac{1}{2}$ gets a payoff $(v_{\mathrm{b}}-\frac{1}{2})$. As noted in the text, a "one-step" equilibrium at price $\frac{1}{2}$ formalizes this outcome.

As described in the text, an equilibrium value of y must satisfy the necessary condition

$$\begin{aligned}(1-y)\,U_{\mathrm{b}}(y;[0,1-y],[y,1])+yU_{\mathrm{b}}(y;[1-y,1],[y,1])\\ =(1-y)\,U_{\mathrm{b}}(y;[0,1-y],[0,y]),\end{aligned} \tag{5}$$

since the left-hand side represents buyer-type y's expected payoff if he says "keen" and the right-hand side represents his expected payoff if he says "not keen."

The first term on the left-hand side of (5) is strictly less than the right-hand side, because the only difference is that in the first term the seller is more optimistic about the buyer's type. Therefore the second term on the left-hand side is strictly positive, so the buyer-type y trades at least sometimes in that subgame ($y>\underline{\beta}=1-\frac{3}{4}y$, or $y>\frac{4}{7}$), and some seller types trade for sure. On the other hand, since in this subgame $\bar{\beta}>1$, not all seller types trade for sure. Thus (3) applies in the second term of (5).

On the right-hand side of (5), which involves the subgame $\{v_{\mathrm{s}}\in[0,1-y],\ v_{\mathrm{b}}\in[0,y]\}$, the critical type $\bar{\beta}$ is equal to $1-\frac{3}{4}y$, so y trades for sure in that subgame, since $y>\frac{4}{7}$ implies $y>\bar{\beta}$. But $\underline{v}_{\mathrm{b}}=0<\underline{\beta}=\frac{1}{4}y$, so no seller type is sure to trade and the bottom case of (2) applies.

Finally, in the first term on the left-hand side of (5), involving the subgame $\{v_{\mathrm{s}}\in[0,1-y],\ v_{\mathrm{b}}\in[y,1]\}$ (in which both players are keen), $\bar{\beta}=\frac{5}{4}-y$. So if $y>\frac{5}{4}-y$, or $y>\frac{5}{8}$, then y trades for sure when both agents are keen. This means that all types of both agents trade for sure, and the Chatterjee–Samuelson analysis breaks down; as discussed in the text, we consider the one-step equilirium in which trade occurs with certainty at price $\frac{1}{2}$.

Substituting all this into (5) yields

$$(1-y)(y-\tfrac{1}{2})+\tfrac{1}{3}y(\tfrac{7}{4}y-1)^2=(1-y)(\tfrac{5}{4}y-\tfrac{1}{2}),$$

which has solutions

$$y=\{22\pm 12\sqrt{2}\}/49 \ = \ 0.103 \text{ or } 0.795.$$

Since the analysis of the second term of (5) proved that $y>\frac{4}{7}$, the solution is $y=0.795$, which exceeds $\frac{5}{8}$, confirming that the Chatterjee–Samuelson equilibrium indeed breaks down when both parties say "keen."

This completes our derivation of the equilibrium value of y. We next state and prove Lemma 1, which describes our equilibrium. Finally, we prove Propositions 1 and 2.

LEMMA 1. *The following behavior defines the equilibrium path of a perfect Bayesian equilibrium in our two-stage bargaining game.* (Our proof does not require explicit descriptions of the optimal bids following deviations in the cheap-talk phase, so we omit these bids from the description of equilibrium.)

Buyer. (A) If $v_b<y$ then

(i) say "not keen" in the first stage, and

(ii) if the seller says "not keen" in the first stage then bid $p_b=0$ in the second stage, but if the seller says "keen" in the first stage then bid

$$p_b=\min\{\tfrac{2}{3}v_b+\tfrac{1}{12}y,\ \tfrac{2}{3}-\tfrac{5}{12}y\}$$

in the second stage.

(B) If $v_b\geqslant y$ then

(i) say "keen" in the first stage, and

(ii) if the seller says "keen" in the first stage then bid $p_b=\frac{1}{2}$ in the second stage, but if the seller says "not keen" in the first stage then bid

$$p_b=\tfrac{2}{3}v_b+\tfrac{1}{4}(1-y)+\tfrac{1}{12}$$

in the second stage.

Seller. (A) If $v_s>1-y$ then

(i) say "not keen" in the first stage, and

(ii) if the buyer says "not keen" in the first stage then bid $p_s = 1$ in the second stage, but if the buyer says "keen" in the first stage then bid

$$p_s = \max\{\tfrac{2}{3}v_s + \tfrac{1}{4} + \tfrac{1}{12}(1-y), \tfrac{1}{3} + \tfrac{5}{12}y\}$$

in the second stage.

(B) If $v_s \leqslant 1 - y$ then

(i) say "keen" in the first stage, and

(ii) if the buyer says "keen" in the first stage then bid $p_s = \frac{1}{2}$ in the second stage, but if the buyer says "not keen" in the first stage then bid

$$p_s = \tfrac{2}{3}v_s + \tfrac{1}{4}y$$

in the second stage.

Proof. The equilibrium bidding strategies in the {"keen," "not keen"} and {"not keen," "keen"} subgames follow from the analysis in the first part of this Appendix. This means that the bidding strategies specified in the lemma are sequentially rational. It therefore suffices to show that the necessary condition (5) is also sufficient. This is a direct result of the fact that higher buyer types are more concerned with the probability of trade than are lower buyer types.

Formally, define the difference between the payoffs from saying "keen" and saying "not keen," given subsequent optimal behavior for a buyer of type v_b, as

$$\begin{aligned} D(v_b) \equiv {} & (1-y)\, W_b(v_b; [0, 1-y], [y, 1]) \\ & + y W_b(v_b; [1-y, 1], [y, 1]) \\ & - (1-y)\, W_b(v_b; [0, 1-y], [0, y]), \end{aligned}$$

where $W_b(v_b; [\underline{v}_s, \bar{v}_s], [\underline{v}_b, \bar{v}_b])$ is the expected payoff to buyer-type v_b from making the optimal bid when the seller's type is uniformly distributed on $[\underline{v}_s, \bar{v}_s]$ and the seller *believes* that the buyer's type is independently and uniformly distributed on $[\underline{v}_b, \bar{v}_b]$. Note that W_b is defined even if v_b falls outside the interval $[\underline{v}_b, \bar{v}_b]$ that the seller believes contains the buyer's type, as will occur off the equilibrium path. If $v_b \in [\underline{v}_b, \bar{v}_b]$, however, then

$$W_b(v_b; [\underline{v}_s, \bar{v}_s], [\underline{v}_b, \bar{v}_b]) = U_b(v_b; [\underline{v}_s, \bar{v}_s], [\underline{v}_b, \bar{v}_b]),$$

as will occur in equilibrium.

We must show that $D(v_b) \geqslant 0$ for $v_b \geqslant y$ and that $D(v_b) \leqslant 0$ for $v_b \leqslant y$. This will show that our proposed equilibrium is indeed an equilibrium—that is, that no buyer can gain by deviating in the cheap-talk stage and then submitting an unexpected bid. (A similar proof works for the seller.) We do this by showing that

(i) for $v_b \leqslant \frac{1}{2}$, $D(v_b) \leqslant 0$,

(ii) for $v_b > \frac{1}{2}$, $D'(v_b) \geqslant 0$.

Since $y > \frac{1}{2}$ and since $D(y) = 0$, (i) and (ii) prove our proposition. Note that the arguments below do not require explicit descriptions of optimal bids following deviations in the cheap-talk phase.

Consider first $v_b \leqslant \frac{1}{2}$. Since the seller bids $\frac{1}{2}$ after {"keen," "keen"}, the first term of $D(v_b)$ vanishes. Moreover, the seller's minimum bid after {"not keen," "keen"} is

$$p_s(1-y) = \max\{\tilde{p}_s(1-y), \tilde{p}_b(y)\} = \tilde{p}_b(y) = \tfrac{1}{3} + \tfrac{5}{12}y,$$

and since $\frac{1}{3} + \frac{5}{12}y > \frac{1}{2}$, the second term of $D(v_b)$ also vanishes. Hence, it is immediate that $D(v_b) \leqslant 0$ for $v_b \leqslant \frac{1}{2}$.

Now consider the derivative $D'(v_b)$, for $v_b \geqslant \frac{1}{2}$. Note that the derivative of the interim payoff is equal to the probability of trade (by the envelope theorem), *even off the equilibrium path.* Since buyer types $v_b \geqslant \frac{1}{2}$ trade for sure in the {"keen," "keen"} subgame (since the seller bids $\frac{1}{2}$ for sure), the derivative of the first term of $D(v_b)$ is $(1-y)$. The derivative of the third (negative) term is $[-(1-y)]$ times a probability. So, for $v_b \geqslant \frac{1}{2}$,

$$D'(v_b) \geqslant \frac{d}{dv_b}\{yW_b(v_b; [1-y, 1], [y, 1])\} \geqslant 0. \quad \text{Q.E.D.}$$

PROPOSITION 1. *The equilibrium mapping from types to outcomes in our cheap-talk equilibrium cannot arise in an equilibrium without talk.*

Proof. Consider a buyer of type $v_b > y$. In equilibrium he sometimes bids $\frac{1}{2}$ (following "keen" from the seller) and sometimes bids

$$\tilde{p}_b(v_b; [1-y, 1], [y, 1]) = \tfrac{2}{3}v_b + \tfrac{1}{4}(1-y) + \tfrac{1}{12} = \tfrac{1}{3} + \tfrac{2}{3}v_b - \tfrac{1}{4}y$$

(following "not keen" from the seller). Note that the latter bid exceeds $\frac{1}{2}$. Dividing his bids into the two classes, "bids $= \frac{1}{2}$" and "bids $> \frac{1}{2}$," we see that the frequency of the first class is $(1-y)$ (i.e., the probability that the seller says "keen"). We notice also that the seller facing this buyer sometimes bids $\frac{1}{2}$ and sometimes bids

$$\begin{aligned} p_s(v_s) &= \max\{\tilde{p}_s(v_s; [1-y, 1], [y, 1]), \tilde{p}_b(y; [1-y, 1], [y, 1])\} \\ &\geqslant \tilde{p}_b(y; [1-, 1], [y, 1]) \\ &= \tfrac{1}{3} + (\tfrac{2}{3} - \tfrac{1}{4})y = \tfrac{1}{3} + \tfrac{5}{12}y > \tfrac{1}{2}. \end{aligned}$$

Given knowledge only of the buyer's type $v_b > y$, therefore, we see a joint distribution of buyer's and seller's bids as

		p_s: $\frac{1}{2}$	p_s: $>\frac{1}{2}$
p_b	$\frac{1}{2}$	$(1-y)$	0
	$>\frac{1}{2}$	0	y

But such correlation in bids would be unattainable in a Nash equilibrium without cheap talk. This shows that the equilibrium *strategies* (maps from types to bids) could not arise without talk.

We now extend this argument to show that the equilibrium mapping from types to outcomes (whether trade occurs and if so at what price) is unattainable in a Nash equilibrium without cheap talk. Consider the buyer types $v_b \in [y, 1]$, who say "keen" in the talk phase. If the seller says "not keen" then trade occurs with positive probability, and if trade occurs then (by Lemma 1) the price moves in v_b. If the seller says "keen," however, then trade occurs with probability one at price $\frac{1}{2}$, which is independent of v_b. Thus, the price at which trade occurs either does or does not depend on v_b according to the value of v_s. Clearly, the equilibrium map from types to outcomes in a double auction without talk cannot behave in this way. Q.E.D.

LEMMA 2. *In any equilibrium without talk, the seller's bid p_s is a weakly increasing function of his type v_s.*

Proof. Given any seller's bid p_s, the distribution $F_b(\cdot)$ of the buyer's bid p_b determines a probability of trade, $\pi = 1 - F_b(p_s)$, and an expected price conditional on trade,

$$e = E_{p_b}\{(p_b + p_s)/2 \mid p_b \geqslant p_s\}.$$

Thus in choosing a bid p_s the seller is choosing a point in (e, π) space. The relationship between p_s and e is strictly monotone, moreover.

In terms of (e, π), the seller's expected payoff is just $\pi(e - v_s)$. Therefore sellers with higher v_s have steeper indifference curves in (e, π) space. This means that for $e' < e$, if v_s prefers (e, π) to (e', π') then so does $v_s' > v_s$.

Let $e^*(v_s)$ be the optimal choice of e on the available locus for seller-type v_s. Then our observation concerning the slopes of the indifference curves for different types implies that e^* is (weakly) increasing in v_s. Consequently, so is the bid p_s. Q.E.D.

PROPOSITION 2. *There does not exist an equilibrium without cheap talk in which sellers $v_s \in [1-y, 1]$ and buyers $v_b \in [y, 1]$ name the prices that they name in the {"not keen," "keen"} subgame of our cheap-talk equilibrium.*

Proof. Suppose the buyer believes that the seller's bid is distributed as $F(p_s)$. Define $Z(v_b, p_b, F)$ to be the expected payoff to buyer-type v_b from bidding p_b when p_s is distributed as F. This buyer-type's optimal bid, $p_b(v_b; F)$, maximizes $Z(v_b, p_b, F)$.

Let F be the buyer's belief about p_s in the {"not keen," "keen"} subgame of our cheap-talk equilibrium. Then from Lemma 1 we have that

$$p_b(v_b; F) = \tfrac{2}{3}v_b + \tfrac{1}{4}(1-y) + \tfrac{1}{12}$$

and that for $v_b \in (y, 1)$ the derivative

$$\left.\frac{\partial Z(v_b, p_b, F)}{\partial p_b}\right|_{p_b(v_b; F)}$$

exists and equals zero.

Now let G be any distribution with support bounded above by $p_s(1-y)$, the lowest seller bid in our {"not keen," "keen"} subgame. A simple computation shows that $p_b(v_b; F)$ can be rewritten as

$$p_b(v_b; F) = p_s(1-y) + (\tfrac{2}{3})(v_b - y),$$

which exceeds $p_s(1-y)$ for $v_b > y$. Since $G[p_s(1-y)] = 1$ and $p_s(1-y) < p_b(v_b; F)$,

$$\left.\frac{\partial Z(v_b, p_b, G)}{\partial p_b}\right|_{p_b(v_b; F)}$$

exists and is strictly negative, for $v_b > y$.

Finally, suppose that there were an equilibrium without talk in which the relevant buyers and sellers name the required prices. Then by Lemma 2, the prices named by sellers $v_s \in [0, 1-y]$ cannot exceed $p_s(1-y)$. The buyer's belief about p_s in such an equilibrium can be described by the distribution

$$H = \lambda F + (1-\lambda) G$$

for some $\lambda \in (0, 1)$ and distributions F and G as described above. Note that

$$Z(v_b, p_b, H) = \lambda Z(v_b, p_b, F) + (1-\lambda)\, Z(v_b, p_b, G).$$

Hence,

$$\left.\frac{\partial Z(v_\mathrm{b}, p_\mathrm{b}, H)}{\partial p_\mathrm{b}}\right|_{p_\mathrm{b}(v_\mathrm{b}; F)} < 0,$$

so $p_\mathrm{b}(v_\mathrm{b}; F)$ is no longer an equilibrium bid for buyer-type $v_\mathrm{b} > y$. Q.E.D.

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