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Arming as a Strategic Investment in a Cooperative Equilibrium

By MICHELLE R. GARFINKEL*

To develop a positive, economic theory of military spending, the analysis focuses on a game-theoretic, general equilibrium model of international conflict, in which consumption, peaceful investment, and military spending are endogenously determined. The analysis illustrates that when there is repeated interaction between nations, a game of threats and punishments generally will not support a disarmament outcome and that fluctuations in military spending can be an endogenous result of fluctuations in aggregate economic activity. Furthermore, the analysis shows how the relation between aggregate economic activity and military spending qualitatively depends on whether governments are acting opportunistically or cooperatively. (JEL 114,321)

This paper develops a positive theory of military spending on armaments. The analysis focuses on a game-theoretic, general equilibrium model of international conflict, in which resource allocation among consumption, peaceful investment, and military spending is endogenously determined. The analysis shows that fluctuations in military spending can be an endogenous result of fluctuations in aggregate economic activity. Accordingly, the analysis raises doubts about existing empirical studies of macroeconomic fluctuations in which military spending is treated as exogenous—for example, see Robert Hall (1986).¹ In addition, taking the repeated interaction between nations explic-

itly taken into account, the analysis shows that the qualitative relation between military spending and aggregate economic activity depends on whether governments are acting cooperatively or opportunistically. Specifically, if governments are acting cooperatively, then the model predicts that, for a given realization of aggregate economic activity, the associated level of military spending will be larger, the larger is the variance of aggregate economic activity and the smaller is the average level of economic activity. Alternatively, if governments were acting opportunistically, then the model predicts that for a given realization of aggregate economic activity, the associated level of military spending would be independent of the average level and the variance of aggregate economic activity.

In contrast to the existing literature on game-theoretic models of international conflict,² the game-theoretic analysis developed

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¹The existing empirical evidence on whether military spending is Granger-causal prior to other aggregate economic variables is mixed. For example, Teresa Garcia-Mila (1987) documents evidence that is consistent with the notion that military spending is statistically exogenous with respect to other aggregate economic variables. But, R. P. Smith (1987) finds that while past values of income and civilian expenditures can help to predict current military spending, past values of military spending do not help predict income and civilian expenditures.

²See Michael Intriligator (1982) for a list of references and Martin Shubik (1987) for a survey of the literature and a critical assessment of the applicability of game-theoretic techniques to the study of the issues in conflict theory. Early applications of the tools of game theory to the study of international conflict, Thomas Schelling (1960) and Intriligator (1975) for example, focus on the study of arms races. Subsequent research—see Dagobert Brito and Intriligator (1984), for example—extends the analysis to investigate the possibility of an arms race leading to a war. Researchers

in the present paper emphasizes two economic aspects of international conflict. First, there are the economic costs incurred by nations who choose to participate in international conflict. The economic costs arise most obviously through a nation's resource constraints. Although the existing analyses include a high degree of mathematical sophistication and address important issues, they are incomplete as they fail to account for resource constraints and, more generally, they abstract from the endogenous determination of resource allocation.

Second, there might be possible economic (among other) motives for nations to confront other nations. Lionel Robbins (1940), in assessing the extent to which war can be regarded as a result of economic motives, suggests as an example that a government acting to promote the interests of its citizens might go to war, whereby, if successful, it could improve its terms of trade and hence economic welfare.³ Although tributes in this century generally have not been significant,⁴

in conflict theory also have created a large literature concerned with retaliation and deterrence. Building on the seminal work of Steven Brams and Donald Wittman (1981), the aim of recent research—for example, see Frank Zagare (1985) and Brams and Marc Kilgour (1985, 1986)—has been to identify and to characterize the nonmyopic equilibria in abstract settings in which the players, interpreted as nations, can make sequential moves conditional on the initial move of their opponent. The equilibrium outcome is finally determined when none of the players has the desire to make a further move. Robert Axelrod (1984) finds that “tit-for-tat” strategies, in which retaliation is possible because of the repeated nature of the game, can explain why there is not a perpetual worldwide war.

³Recently, Tom Riddell (1988), who recognizes the importance of the link between a nation's international economic objectives and its military spending, documents a positive relation between U.S. military power and its economic welfare in the post-World War II period.

⁴Even in cases where nations do not appear to be better off as a result of wartime activities, it is not clear that these nations did not participate in hope of obtaining tribute. However, see William Shepherd (1988), who provides an interesting discussion of the relevance of the tribute as a motivation for military spending as a defensive as well as an offensive measure. As he notes, there have been instances of large tributes in this century. For example, Alan Milward (1970, p. 273) calcu-

lates that payments to Germany from France as a percentage of French national income in 1938 at 1938 prices from 1940 through the first six months of 1944 were respectively 10.9, 19.3, 20.9, 36.6, and 27.6—an average of 23 percent of 1938 French national income per year. But, taking into account the costs of the German war machine (even excluding casualties) and those of occupation, tends to diminish the relevance of the tribute.

victories in confrontations historically have generated substantial tributes (especially for nations confronting weaker neighboring nations).⁵ Nevertheless, economists who have examined the game-theoretic aspects of economic policymaking in interdependent economies—see, for example, Matthew Canzoneri and Jo Anna Gray (1985) and Canzoneri and Dale Henderson (1986, 1987)—abstract from war, or a threat thereof, as a possible strategy to promote the economic interests of the nation. Dagobert Brito (1972), who develops a two-nation, general equilibrium model of an armaments race and accounts for the

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⁵Whether or not the ability to extract or to avoid paying tribute provides a sufficient motivation to endure the costs of producing armaments is a matter of controversy. Indeed, it seems reasonable to argue that most modern conflicts between nations are driven not by booty, but by differences in religion or ideologies, for example. Although such differences are important in international conflicts, however, it is not clear that they serve as the primary driving force for confronting other nations. For example, following Douglas North (1981), who argues that ideologies are crucial for circumventing free-rider problems, one could argue that ideological differences might emerge as a government's justification to build its military strength. Without ideologies, individual citizens would not willingly contribute to the nation's mobilization efforts, since it is not feasible for the government to make the individual's return on the military investment as a function of his own contribution. Hence, while differences in ideologies might appear to be at the root of most modern international conflicts, such differences actually might emerge as a by-product of a nation's mobilization efforts to extract tribute from other nations or avoid paying tribute to stronger nations. Alternatively, tribute could be thought of as the gain in utility generated by the success of a nation's wartime efforts, though the actual transfer of goods might be negligible or even negative. The analysis to follow could incorporate such a notion of tribute. Provided that consumption goods and the success in an ideological or religious dispute are relatively close substitutes, the qualitative results to follow would hardly change.

costs of such a race through a nation's resource constraints, does not identify the benefits of such a race. Instead, he simply includes defense expenditures by both nations in individuals' utility function, presumably to capture the notion that such expenditures, holding the enemy's expenditures fixed, generate psychic benefits.⁶ Joshua Feinman, Peter Garber, and Michelle Garfinkel (1986) develop a general equilibrium model in which nations arm in order to advance the economic interests of their citizens, who do not directly derive utility from armaments. The result in this model that economic motives lead to an arms race is not general, however, as this model assumes that the governments play a one-shot game.

The model in this paper indirectly builds on an economic motivation for investing in armaments, but formalizes the notion that modern conflicts, as reflected in a nation's military spending, might not be directly about tribute. Success in a confrontation, which depends on the relative amounts of armaments each nation constructs, would generate goods for future consumption. As in Feinman, Garber, and Garfinkel (1986), without repeated interaction between the players, in which case governments act non-cooperatively by taking the opponent's strategy as given, a significant amount of resources would be wasted away in military spending. Although no transfer of goods would be observed in this hypothetical outcome, each nation's choice to endure the costs of arms production would be driven by the possibility of extracting or avoiding extraction of resources. In the one-shot game, a commitment to peaceful policies, with perhaps a voluntary transfer of goods, is not an equilibrium strategy. Specifically, because each government would choose to deviate from the commitment policy by producing armaments, disarmament is not a subgame perfect equilibrium.

⁶See also F. van der Ploeg and A. J. de Zeeuw (1986a, b). Such a formulation might be made consistent with the idea mentioned above that tribute could be the gain in utility from a successful confrontation driven by ideological or religious differences.

With repeated interaction between sovereign states, however, threats and punishments can support cooperative behavior.⁷ The resulting "cooperative equilibrium," where cooperation is not assumed but derived as an optimal strategy, corresponds to what is referred to as the "trigger strategy equilibrium" in the game theory literature. In the cooperative equilibrium, disarmament is a possibility. More generally, the amount of resources allocated to military spending is positive, possibly with intermittent periods of positive and zero military spending on armaments. But, for each play of the game, military spending is less than in the one-shot game.

Military spending in the cooperative equilibrium does not arise directly from an economic motive, as in the one-shot game without commitments. Instead, the positive production of armaments arises from a deterrence motive, for it is necessary to induce cooperative behavior.

In what follows, the next section presents a modest extension of the economic model of international conflict developed in Feinman et al. (1986) and examines the characteristics of the equilibria that would emerge if the governments were to act "opportunistically" (the opportunistic equilibrium) and, alternatively, if governments were able to commit themselves to disarmament (the efficient outcome). Section II examines the role of threats and punishments in the cooperative equilibrium, identifying the conditions under which the strategic interplay between governments would be sufficient to support

⁷Mancur Olson and Richard Zeckhauser (1966), who analyze a model of alliances to formulate a theory of military spending that can explain the fact that larger members typically devote a larger proportion of available resources to military spending than smaller members, assume that members of the alliance act opportunistically. They implicitly assume, however, that there is sufficient cooperation for the existence and stability of the international organization. Moreover, their analysis ignores the optimizing behavior of nonmember nations, and in particular, the influence that enemy nations' military spending has on the effectiveness of member nations' military spending in providing defense for the alliance.

the (otherwise time-inconsistent) disarmament solution. Furthermore, this section characterizes military spending in the cooperative equilibrium when threats and punishments are not sufficient to support disarmament, and derives the qualitative differences between military spending in the opportunistic and the cooperative equilibria. The qualitative differences, in turn, imply some testable hypotheses to determine whether governments are acting opportunistically or cooperatively. Finally, Section III summarizes the results.

I. Analytical Framework

This section presents a simple two-sovereign state model of international conflict. As in most analyses of international conflict, this analysis takes the political structure as given. That is, it is assumed that the problems of coordination cannot be resolved by combining the two states into one. The model is essentially an infinitely repeated game version of the two-period war game originally discussed in Feinman et al. (1986). In particular, the simple two-sovereign state, two-period, general equilibrium model discussed in that paper is placed in a framework in which infinitely lived governments maximize the utility of their citizens who are also infinitely lived. Individuals in the domestic state and the foreign state, denoted by “*,” have identical tastes, technologies, and endowments. Every two periods, $t = 0, 2, 4, \dots, \infty$, each individual receives an endowment of Z_t units of non-storable goods, which he allocates to current consumption, c_t , and to the production of goods to be consumed in the next period, c_{t+1} . Although Z_t is realized before individuals make their decisions in period t , future values of the endowment are uncertain. The endowment is generated by a stationary stochastic process, whose distribution is given by $F(Z)$ and $\int Z_t^h dF(Z) = 1$. Let \bar{Z} denote the expected value of the endowment and σ^2 denote the variance of the endowment.

Individuals maximize expected lifetime utility, which depends on consumption in all periods of their lives. The unconditional expectation of a representative individual's

lifetime utility is given by

$$(1) \quad E\{U_0\} = \sum_{t=0}^{\infty} \int_{Z_t} Z_t^h \beta^t u(c_t) dF(Z),$$

$$0 < \beta < 1,$$

where β reflects the subjective rate of time preference and $E\{\cdot\}$ denotes the unconditional expectations operator. $u(c_t)$ satisfies $u'(c_t) > 0$, $u'(0) = \infty$, and $u''(c_t) < 0$.

There are two ways in which the endowment good can be transformed into goods for future consumption.⁸ The first method is a peaceful investment technology, $f(i_t)$, with $f(0) = 0$, $f'(i_t) > 0$ and $f''(i_t) < 0$, enabling an individual to transform i_t units of his endowment received in t into $f(i_t)$ goods for consumption in the next period, $t+1$, for $t = 0, 2, 4, \dots, \infty$.

The second method of transferring endowment goods received in t to $t+1$ involves military spending. A unit of the endowment good can be converted into a unit of war goods, W_t , which is also non-storable.⁹ The possibility of extracting resources from an enemy state can serve as a motivation for producing war goods (or equivalently armaments).¹⁰ In this case, although war goods

⁸For simplicity, it is assumed that these methods, which generate a certain return, are only available in the periods in which the endowment is received. While the qualitative results do not hinge on this simplification, this assumption, in conjunction with the assumptions that $Z_{t+1} = 0$ and that goods are non-storable (so that there is no accumulation of capital stocks), is important for it strips the model of any dynamics that would otherwise obscure the dynamics introduced by strategic considerations. Also, the analysis abstracts from labor as a factor of production in both methods. Including labor would not change the analysis to follow substantially.

⁹Ruling out the possibility of building up a capital stock of armaments with military spending makes the model tractable, and, as noted above, permits the analysis to highlight the dynamics arising from strategic considerations.

¹⁰However, as shown below in Section II, in the cooperative equilibrium, such a possibility serves only as an indirect motive for producing war goods. Specifically, the primary motivation is to induce cooperation by reducing each nation's incentive to act opportunistically in an effort to extract resources from the enemy nation.

do not directly yield utility to individuals, they can do so indirectly, as success or failure in the confrontation results in an increase or a decrease in goods for consumption in the next period, through the imposition of a tribute on the weaker nation.

The outcome of a confrontation, which is reflected in the amount of tribute one nation extracts from the other nation, depends on the relative amounts of war goods constructed by the two nations. Specifically, tribute per citizen of the domestic nation in $t + 1$, which can be positive or negative, is determined by:

$$(2) \quad \Gamma(W_t, W_t^*) = \begin{cases} a(W_t - W_t^*), & \text{if } -f(i_t) \leq a(W_t - W_t^*) \leq f(i_t^*); \\ f(i_t^*), & \text{if } a(W_t - W_t^*) > f(i_t^*); \\ -f(i_t), & \text{if } a(W_t - W_t^*) < -f(i_t), \end{cases}$$

for $t = 0, 2, 4, \dots, \infty$. The expression in (2) says that the amount of tribute extracted depends linearly on the difference in the amount of war goods constructed by the two nations, but it cannot exceed the amount of goods available to the weaker state in $t + 1$. The marginal product of war goods, a , is strictly positive. As noted below, a stable equilibrium requires $a < f'(0)$.

Different magnitudes of a gives rise to different possible interpretations of the tribute function.¹¹ If, for instance, a is very large, then the expression in (2) would capture the case in which one nation, having

one more armament than the opponent, could extract the opponent's entire $t + 1$ return on the peaceful investment.¹² Alternatively, if a is not very large, then the tribute function can be thought of as the result of a bargaining process between the two nations. War goods, then, enhance each nation's bargaining power according to (2). For example, in a more elaborate model, the weaker nation would be forced to coordinate its macroeconomic policy in a way that improves the economic welfare of the stronger nation and simultaneously deteriorates the economic welfare of the weaker nation.

In any case, (2) assumes that the outcome of the confrontation simply involves a comparison of the amounts of war goods constructed by each nation. Note that the expression in (2) implies that the returns from military spending for the domestic and the foreign states must sum to zero—that is, $\Gamma(W_t, W_t^*) = -\Gamma^*(W_t^*, W_t)$.¹³

Assume that each of the infinitely lived governments of the two nations gains the resources necessary to play the war game through lump sum taxation of its citizens in t and distributes or raises the tribute through lump sum transfers or taxes in $t + 1$ for $t = 0, 2, 4, \dots, \infty$. Assume also that governments' policies are formulated in the interests of the representative individual.¹⁴ Accordingly, at the beginning of every two

¹²The value of a that would permit such an interpretation can be finite.

¹³One possible extension of the model that would permit a clear distinction between mobilization and actual warfare would involve assuming that the endowment can differ *ex post* across nations and that both technologies for transferring goods into the future are stochastic. In particular, suppose that the return from the peaceful investment were realized in $t + 1$, for $t = 0, 2, \dots, \infty$, and could differ *ex post* across nations. At that time, after mobilization decisions have been made, each nation decides whether or not to initiate a war, the return from which is uncertain. The variable that would capture the uncertainty associated with fighting, in the form of an additive term in the $t + 1$ resource constraint (see equation (3b)), would be identically zero if there were no fighting and strictly negative for each nation initiating a strike. In this extension, involving a two-stage maximization problem, warfare would not be a zero sum game.

¹⁴Principal-agent problems are ignored.

¹¹See Shepherd (1988) who describes the possible technological and institutional factors that determine the magnitude of the parameter a . Another interpretation of the conflict as embedded in (2), would arise in a nonnation player setting. For example, the players might be pirates or thieves. With minor modifications, the model also might apply to the game of contestable transfers, where groups lobby in the political arena to change the existing policy. A group's failure to participate could be costly if that group would have to help to finance that transfer to the winner.

periods, each government selects the magnitude of military spending, W_0 , and plans a sequence of future contingent levels of military spending, $\{W_t\}_{t=2,4}^\infty$ to maximize the expected utility of the representative individual. The domestic government maximizes the objective function given by (1) subject to (2) and the following constraints:

$$(3a) \quad c_t = Z_t - i_t - W_t$$

$$(3b) \quad c_{t+1} = f(i_t) + \Gamma(W_t, W_t^*),$$

for $t = 0, 2, 4, \dots, \infty$. The foreign government solves an analogous problem.

Parts A and B study the opportunistic and the efficient solutions, which differ with respect to the specification of a government's strategy and perception of its opponent's strategy. These hypothetical solutions serve as benchmark solutions to which the cooperative equilibrium, studied below in Section II, can be compared. In the analyses to follow, it is assumed that the informational structure of the model is perfect in the sense that each government knows the opponent's optimization problem and the constraints (including the current realization of the other state's endowment) faced by the opponent. In addition, it is assumed that the governments act simultaneously. This assumption is sufficient to ensure perfection in the one-shot game.

A. The Opportunistic Equilibrium

This section assumes that each government solves its problem of maximizing the utility of the representative agent (1), taking its opponent's military spending as given. The resulting equilibrium—the opportunistic equilibrium—corresponds to the subgame perfect Nash (noncooperative) equilibrium in the infinite horizon game, assuming that a government's strategy is such that its actions are in no way contingent on the previous actions of the opponent. Such an equilibrium is unsatisfactory, for it effectively assumes that the governments do not recognize the impact that their current policy can have on their opponents' future policies and hence on future outcomes. Such an

assumption, which would be realistic only if individuals and governments live for two periods, leads to the equilibrium studied by Feinman et al. (1986).

At the beginning of every two periods, $t = 0, 2, \dots, \infty$, each government chooses levels of peaceful investment and military expenditures to maximize the utility of the representative agent (1), subject to the tribute function (2) and the resource constraints (3), given the current realization of the endowment and taking the opponent's strategy as given. The assumptions imposed on the instantaneous utility function, $u(c_t)$, and the peaceful investment technology, $f(i_t)$, imply that the solution to the governments' problem must satisfy the following first-order condition:

$$(4a) \quad \frac{u'(c_t)}{u'(c_{t+1})} = \beta f'(i_t),$$

for $t = 0, 2, \dots, \infty$, where $f'(0) > a$.¹⁵ In addition, assuming an interior solution for military spending in period t , which implies that $f'(Z_t) < a$, the solution must satisfy the following first-order condition:

$$(4b) \quad f'(i_t) \leq a,$$

for $t = 0, 2, \dots, \infty$. The expression in (4b) possibly results in a strict inequality condition because of the truncation of the tribute function.

In order to derive specific functional forms for the reaction functions, which relate a nation's military spending to that of the opponent, the following specifications for utility and the peaceful investment technol-

¹⁵If $f'(0) \leq a$, then peaceful investment in period $t = 0$ would be zero for both nations, so that investment in armaments would be fruitless. But, given that the opponent produces zero armaments, each government would be better off by investing some fraction of the endowment in the peaceful technology, thereby giving rise to the opponent's incentive to shift resources from the peaceful technology to the production of armaments. The oscillating nature of this reaction process suggests that a stable Nash equilibrium does not exist in the one-shot game when $f'(0) \leq a$.

ogy are made:

$$(5a) \quad v_t = \ln(c_t) + \beta \ln(c_{t+1})$$

$$(5b) \quad f(i_t) = Ai_t^\alpha, \quad 0 < \alpha < 1,$$

for $t = 0, 2, 4, \dots, \infty$, where v_t denotes the representative agent's two-period utility starting in period t .¹⁶ Given this specification, the first-order conditions from choosing optimal investment and military expenditures, given by (4), imply the following reaction function for the domestic government:

$$(6) \quad \bar{W}_t(W_t^*) = \frac{\beta}{1+\beta} Z_t \\ - \frac{1+\alpha\beta}{\alpha(1+\beta)} \left(\frac{\alpha}{a} A \right)^{\frac{1}{1-\alpha}} \\ + \frac{1}{1+\beta} W_t^*,$$

for $t = 0, 2, 4, \dots, \infty$. The reaction function for the foreign government is found by simply replacing W_t^* with W_t . As is evident from (6), a nation's military spending is positively related to that of its opponent, but this relationship is not one for one. Generally speaking, provided that consumption goods, c_t and c_{t+1} , are normal, an increase in an opponent's military spending will induce a nation to increase its own military spending, but by less than the increase in the opponent's spending.

If $Z_t > ((1+\alpha\beta)/\alpha\beta)(\alpha A/a)^{1/(1-\alpha)} \equiv k$, as assumed throughout the analysis below, each period $t = 0, 2, \dots, \infty$, both governments will construct positive amounts of war goods. By combining the domestic government's reaction function (6) with that for the foreign government, noting that the assumption of identical nations implies $\bar{W}_t = \bar{W}_t^*$,

¹⁶Note that this specification for the peaceful investment technology ensures that $f'(0) > a$ for any finite value of a , such that the analysis to follow is robust to the different interpretations of the tribute function suggested above. Furthermore, note that the specifications made in (5) can be thought of as an approximation to a more general set of functions.

and using (3a), (3b), and (5), one can establish that the solutions to this game are given by

$$(7a) \quad \bar{W}_t = Z_t - k$$

$$(7b) \quad i_t = \frac{\alpha\beta}{1+\alpha\beta} k$$

$$(7c) \quad c_t = \frac{1}{1+\alpha\beta} k$$

$$(7d) \quad c_{t+1} = \frac{\alpha\beta}{1+\alpha\beta} k,$$

for $t = 0, 2, 4, \dots, \infty$. In addition to there being a stalemated confrontation, symmetry implies that peaceful investment and consumption are identical in the foreign state. In this equilibrium, the utility of the domestic government is given by the following:

$$(8) \quad U_0^n = \frac{1}{1-\beta^2} \left((1+\beta) \ln \frac{k}{1+\alpha\beta} + \beta \ln \alpha\beta \right)$$

and symmetry implies that $U_0^n = U_0^{n*}$.¹⁷

Because war goods do not directly yield utility to individuals, the amount of resources allocated to military spending represents a deadweight loss arising from the international conflict. In the opportunistic equilibrium, \bar{W}_t goods are essentially thrown

¹⁷As indicated by (7a), provided that the endowment is sufficiently large, any increase in the endowment above k is allocated to military spending. This result depends on the constancy of the marginal product of war goods. Even if the marginal product of war goods were diminishing, some positive fraction of the hypothetical increase in the endowment would be allocated to the war effort. The marginal product of war goods is assumed to be constant to highlight the tradeoff between peaceful and military investment, and thereby the impact of the strategic behavior of governments to be studied below in Section II.

away in periods $t = 0, 2, \dots, \infty$ and any increase in the endowment above k is simply wasted away in military spending.

B. The Efficient Outcome

In the section above, the equilibrium concept—that is, the one-shot, Nash equilibrium—effectively assumes that governments ignore the potential impact that they can have on the enemy government's policies. In contrast, this section imagines that governments are able to coordinate their policies and to effectively commit themselves to current and future policies regarding military spending. In such a scenario, disarmament prevails, and the waste of resources is eliminated: $W_t = W_t^* = 0$ for all t .

The efficient outcome is the solution to the maximization of (1), subject to (3), given the current realization of the endowment, Z_0 , and $W_t = W_t^*$ for all t . The first-order condition (4b) becomes irrelevant. The solutions in the efficient outcome are given by the following:

$$(9a) \quad i_t = \frac{\alpha\beta}{1 + \alpha\beta} Z_t$$

$$(9b) \quad c_t = \frac{1}{1 + \alpha\beta} Z_t$$

$$(9c) \quad c_{t+1} = A \left(\frac{\alpha\beta}{1 + \alpha\beta} Z_t \right)^\alpha,$$

for $t = 0, 2, 4, \dots, \infty$. As indicated by (9), $W_t = W_t^* = 0$. Symmetry implies again that consumption and investment in the foreign state are also given by (9). Consequently, the unconditional expected utility for the domestic government, which is identical to that for the foreign government, is given by the following:

$$(10) \quad E\{U_0^c\} = \sum_{t=0,2}^{\infty} \int_{Z_t}^{Z_h} \beta^t \left((1 + \alpha\beta) \ln \frac{Z_t}{1 + \alpha\beta} + \beta \ln A + \alpha\beta \ln \alpha\beta \right) dF(Z).$$

By subtracting (8) from (10), one can evaluate the expected difference between the utility obtained in the efficient solution and that obtained in the opportunistic equilibrium:

$$(11) \quad E\{U_0^c - U_0^n\} = \sum_{t=0,2}^{\infty} \int_{Z_t}^{Z_h} \beta^t (1 + \alpha\beta) \ln \frac{Z_t}{k} > 0.$$

The expected difference is strictly positive if the endowment, Z_t , is sufficiently large to ensure positive military expenditures in the opportunistic equilibrium for periods $t = 0, 2, \dots, \infty$.

Without repeated interaction between governments, a third party or some institution to enforce commitments, neither state could avoid the temptation to abrogate the commitment to zero military spending. The domestic government, for example, would have an incentive not to cooperate by constructing a positive amount of war goods according to the reaction function (6), $\bar{W}(W^* = 0, Z_t)$. Of course, the foreign government would have the same incentive. In other words, in the one-shot game, the absence of enforceable commitments to disarmament precludes the feasibility of the disarmament outcome. However, as demonstrated in the next section, with repeated interaction, threats and punishments might be able to support the disarmament outcome.

II. Threats, Punishments, and Cooperation

This section contains an examination of strategic considerations which can substitute, at least to some degree, for each government's inability to make commitments. The fundamental idea here is that when nations repeatedly interact, each government can indirectly influence its opponent's strategy with threats and punishments for deviant behavior, without having to rely on commitments. Threats and punishments create a link between a government's past policy and its opponent's current policy. This link provides the mechanism whereby a government can influence its opponent's future policies through its own current actions. If,

for example, the foreign government cooperates, by pursuing the endogenously determined cooperative policy, then the domestic government will implement the cooperative policy in the next play of the game. If, however, the foreign government acts opportunistically, then the domestic government imposes some punishment, the threat of which must be credible. Because the threat of punishment, triggered by opportunistic behavior, serves as sufficient motivation for both governments to maintain cooperation, the punishment is never observed in the equilibrium.

The cooperative equilibrium considered in this section assumes that the governments employ the trigger strategy originally studied by James Friedman (1971). Trigger strategies, which differ with respect to the length of the punishment, specify that the punishment for deviating from the endogenously determined cooperative policy involves a reversion to the opportunistic solution. While such trigger strategies, as an expectations mechanism, might be realistic in models of monetary policy, such as Robert Barro and David Gordon (1983) and Herschel Grossman and John Van Huyck (1986), where the government is playing a game against atomistic agents, limiting consideration to trigger strategies in the context of this model be questionable. Specifically, as demonstrated by Dilip Abreu (1986), if no trigger strategy is sufficiently severe to support the peaceful outcome, then more severe punishment strategies could be considered. Presumably, governments acting optimally announce and follow strategies that involve the most severe, credible threat, because the corresponding punishment strategy will lead to the best outcome possible. As argued at the end of this section, however, the result that threats and punishments cannot always support the disarmament solution does not depend on the assumption that the governments announce and follow a trigger strategy.

Assume that in period $t = 0$, the domestic government expects the foreign government to follow a contingent cooperative policy, denoted by $\tilde{W}(Z_t)$, for $t = 0$ and so announces that it too will follow this contingent cooperative policy in $t = 0$. Further-

more, the domestic government announces that it will continue to follow this contingent cooperative policy for $t = 2, 4, \dots, \infty$, provided that the opponent has never deviated from that policy. The domestic government threatens to act opportunistically forever if the foreign government ever acts opportunistically. Specifically, the domestic government announces the following strategy:

$$W_0 = \tilde{W}(Z_0),$$

$$(12) \quad W_t = \begin{cases} \tilde{W}(Z_t), & \\ \text{if } W_s^* = \tilde{W}^*(Z_s) & \\ \text{and } W_s = \tilde{W}(Z_s), & \\ 0 \leq s < t; & \\ \bar{W}_t(\bar{W}_t^*, Z_t), & \\ \text{otherwise,} & \end{cases}$$

for $t = 2, 4, \dots, \infty$.¹⁸ The foreign government announces a similar strategy. According to this strategy, the punishment period for opportunistic behavior is infinite. While a shorter punishment could be negotiated by the two governments, the strategy given by (12) is subgame perfect, for it is simply the best one-shot, optimal response.¹⁹ Moreover,

¹⁸In the extension of the model, briefly described in fn. 13, to permit a distinction between mobilization and warfare, the deterrence effect of military spending embedded in the trigger strategy could be twofold. First, as in the present model, it would reduce the incentive to build a war machine in an ultimate effort to extract tribute. Second, given each nation's mobilization efforts and the realization of the return on the peaceful investment (both of which can differ across nations), military spending by each nation would reduce the incentive of the enemy to engage in a preemptive strike by providing the armaments for a retaliatory strike by the potentially preempted nation.

¹⁹One possible objection that can be made against the trigger strategy, as well as the strategies studied by Abreu (1986), revolves around the possibility of future renegotiation. Specifically, the subgame perfection property does not preclude the possibility that once one government has deviated from the cooperative policy, both governments might be able to agree to skip the punishment phase of the strategy and start the game over. If both governments recognize this possibility, then a new situation arises and the credibility of the punishment is questionable. One potential way to treat this problem is to model the probability of successful

among the whole set of trigger strategy equilibria, the solution that involves an infinite punishment period supports the best outcome. Thus, the trigger strategy described in (12) is optimal within the class of trigger strategies.

The problem for the two governments in period $t=0$ is to choose $\tilde{W}(Z_0)$ and the contingent cooperative policy, $\tilde{W}(Z_t)$ for $t=2,4,\dots,\infty$, so as to maximize the expected utility of their representative citizens (1), subject to (3) and (12). The choice of the contingent cooperative policy defines the cooperative equilibrium, in which for all possible realizations of the endowment, Z_0 , neither government has an incentive to act opportunistically in period $t=0$ nor expects to have such an incentive in periods $t=2,4,\dots,\infty$. The optimal cooperative policy, $\tilde{W}(Z_t)$, is a member of the set of possible cooperative solutions. As a member of this set, $\tilde{W}(Z_t)$ must satisfy the following subgame perfection constraint for all possible values of Z_0 :

$$(13) \quad v_0(\bar{W}(\tilde{W}^*(Z_0)), \tilde{W}^*(Z_0)) \\ - v_0(\tilde{W}(Z_0), \tilde{W}^*(Z_0)) \\ \leq \beta^2 E \{ \tilde{U}_2 - U_2^n \}$$

where

$$v_i(W, W^*, Z) = v_i(c_i(W, Z), \\ c_{i+1}(W - W^*, Z)), \\ E \{ \tilde{U}_2 \} = \sum_{i=2,4}^{\infty} \int_{Z_i}^{Z_h} \beta^{i-2} \\ \times v_i(\tilde{W}(Z_i), \tilde{W}^*(Z_i), Z_i) dF(Z_i), \\ E \{ U_2^n \} = \sum_{i=2,4}^{\infty} \int_{Z_i}^{Z_h} \beta^{i-2} \\ \times v_i(\bar{W}(Z_i), \bar{W}^*(Z_i), Z_i) dF(Z_i),$$

renegotiation, in the case that one government has behaved opportunistically, as an exogenous stochastic process. (See Grossman and Van Huyck (1988) who similarly model the memory of private agents as a stochastic process)

and $\bar{W}(\tilde{W}^*(Z_0), Z_0)$ denotes the opportunistic amount of military spending by the domestic government, given that the foreign government is cooperating in period $t=0$. A similar condition must hold for the foreign nation. Given the stationarity of the stochastic process generating Z_t , the solution, $\tilde{W}(Z_t) = \tilde{W}^*(Z_t)$, that satisfies (13), will satisfy a condition analogous to (13) for $t=2,4,\dots,\infty$.

The left-hand side of (13) is equal to the "temptation" to not cooperate. It is the difference in the two-period utility obtained when acting opportunistically and that obtained when implementing the cooperative policy. The temptation will be strictly positive for $\tilde{W}(Z_0) < \bar{W}(\bar{W}^*, Z_0)$. The right-hand side of the inequality (13) is the expected difference between utility obtained when implementing the cooperative policy and that obtained when implementing the opportunistic policy starting in $t=2$. In other words, it is government's expected gain in utility from continuing to cooperate and foregoing the immediate gain that could be realized by failing to cooperate in period $t=0$. In contrast to the temptation, which depends on the current realization of the endowment, the expected gain is fixed. The subgame perfection constraint (13) requires that the fixed expected gain exceed the temptation for all possible realizations of the endowment.

There is at least one solution ($\tilde{W}(Z_t)$, $\tilde{W}^*(Z_t)$) that satisfies (13)—namely, the opportunistic solution, which makes both sides of the inequality in (13) equal to zero. Depending on the parameters of the model, including the distribution of the endowment, the peaceful investment technology, the tribute function, and the discount factor, the set of possible cooperative equilibria, which will generally have more than one element, might include the disarmament solution. In any case, given that the governments are following the trigger strategy specified in (12), because the amount of resources allocated to military spending in cooperative equilibrium represents a deadweight loss, the optimal contingent cooperative policy, $\tilde{W}(Z_t)$, will specify the smallest, incentive compatible amount of military spending for each real-

ization of the endowment—that is, it will equal the smallest $\tilde{W}(Z_t)$ that satisfies (13).

A. Incentive Compatible Military Spending

In the example in which preferences and the peaceful investment technology are given by (5), the trigger strategy generally does not serve as a perfect substitute for the ability to make commitments. To see that the disarmament solution is not necessarily sustained in a trigger strategy equilibrium, it is helpful to calculate the temptation to deviate from the cooperative policy and the gain from maintained cooperation for the cooperative policy $\tilde{W}(Z_t)$.

Given that the foreign government cooperates in $t = 0$, the opportunistic behavior by the domestic government, using the reaction function (6), the first-order conditions (4) and the budget constraints (3), are given by the following:

$$(14a) \quad \bar{W}(\tilde{W}(Z_0), Z_0) = \frac{\beta}{1+\beta}(Z_0 - k) + \frac{1}{1+\beta}\tilde{W}(Z_0)$$

$$(14b) \quad i_0 = \left(\frac{\alpha}{a}A\right)^{\frac{1}{1-\alpha}}$$

$$(14c) \quad c_0 = \frac{1}{1+\beta} \times \left(Z_0 + \frac{1-\alpha}{\alpha}\left(\frac{\alpha}{a}A\right)^{\frac{1}{1-\alpha}} - \tilde{W}(Z_0)\right)$$

$$(14d) \quad c_1 = \frac{\alpha\beta}{1+\beta} \times \left(Z_0 + \frac{1-\alpha}{\alpha}\left(\frac{\alpha}{a}A\right)^{\frac{1}{1-\alpha}} - \tilde{W}(Z_0)\right),$$

where, as defined previously,

$$k \equiv \frac{1+\alpha\beta}{\alpha\beta}\left(\frac{\alpha}{a}A\right)^{\frac{1}{1-\alpha}},$$

the threshold level of Z above which there is military spending in the opportunistic equilibrium. The solution (14) assumes that the magnitude of the opportunistic level of military spending does not trigger the kink in the tribute function (2).²⁰ Such opportunistic behavior yields the following two-period utility:

$$(15) \quad v_0(\bar{W}(\tilde{W}(Z_0)), Z_0 - \tilde{W}(Z_0)) = (1+\beta)\ln\left(Z_0 + \frac{1-\alpha}{\alpha}\left(\frac{\alpha}{a}A\right)^{\frac{1}{1-\alpha}} - \tilde{W}_0(Z)\right) \frac{1}{1+\beta} + \beta \ln \beta a.$$

Under the cooperative policy, both governments construct $\tilde{W}(Z_0)$. They maximize the representative agent's utility (1) with respect to peaceful investment, subject to (3) and $W_t = \tilde{W}Z_t$ for $t = 0, 2, \dots, \infty$. By using (4a) and the constraints in (3), one can easily verify that the optimal levels of consumption and the peaceful investment, for the cooperative policy, \tilde{W} , are given by

$$(16a) \quad i_t = \frac{\alpha\beta}{1+\alpha\beta}(Z_t - \tilde{W}(Z_t))$$

$$(16b) \quad c_t = \frac{1}{1+\alpha\beta}(Z_t - \tilde{W}(Z_t))$$

$$(16c) \quad c_{t+1} = A\left(\frac{\alpha\beta}{1+\alpha\beta}(Z_t - \tilde{W}(Z_t))\right)^\alpha,$$

²⁰That is to say, the following condition must hold:

$$\frac{\alpha\beta}{1+\beta}(Z_t - k - \tilde{W}(Z_t)) \leq A\left(\frac{\alpha\beta}{1+\alpha\beta}(Z_t - \tilde{W}(Z_t))\right)^\alpha,$$

for $t = 0$. In the case that this condition is not satisfied, the opportunistic behavior for the construction of war goods is on the kink of the tribute function,

$$\bar{W}(\tilde{W}(Z_t), Z_t) = \frac{A}{a}\left(\frac{\alpha\beta}{1+\alpha\beta}(Z_t - \tilde{W}(Z_t))\right)^\alpha.$$

The values of the optimal consumption and investment become complicated without providing any additional insight into the nature of the cooperative solution.

for $t = 0, 2, 4, \dots, \infty$. Such cooperative behavior yields the following two-period utility:

$$(17) \quad v_0(\tilde{W}(Z_0), \tilde{W}(Z_0)) \\ = (1 + \alpha\beta) \ln \frac{Z_0 - \tilde{W}(Z_0)}{1 + \alpha\beta} \\ + \beta \ln A + \alpha\beta \ln \alpha\beta.$$

The difference between (15) and (17) represents the domestic government's temptation to deviate from the cooperative policy, contingent on the current realization of the endowment, given that the foreign government is cooperating. The temptation, which is an increasing function of the difference between the level of the endowment and the amount of cooperative military spending, equals zero for $\tilde{W}(Z_0) = \bar{W}(\bar{W}^*, Z_0)$ and is strictly positive for $\tilde{W}(Z_0) < \bar{W}(\bar{W}^*, Z_0)$. Letting $T(Z_0 - \tilde{W}(Z_0))$ denote the temptation to deviate from the cooperative policy, it is easy to establish that

$$\frac{\partial T(Z_0 - \tilde{W}(Z_0))}{\partial \tilde{W}(Z_0)} \\ = - \frac{(1 + \beta) \frac{\alpha}{1 - \alpha}}{\frac{\alpha}{1 - \alpha} (Z_0 - \tilde{W}(Z_0)) + \left(\frac{\alpha}{a} A\right)^{\frac{1}{1 - \alpha}}} \\ + \frac{1 + \alpha\beta}{Z_0 - \tilde{W}(Z_0)}.$$

For $\tilde{W}(Z_0) < \bar{W}(\bar{W}^*, Z_0)$, an increase in the amount of cooperative war expenditures decreases the temptation to cheat, thereby serving as a deterrent against opportunistic behavior. Furthermore, it is easy to verify by differentiating the temptation with respect to Z_0 , while holding \tilde{W}_0 fixed, that

$$\frac{\partial T(Z_0 - \tilde{W}(Z_0))}{\partial Z_0} = - \frac{\partial T(Z_0 - \tilde{W}(Z_0))}{\partial \tilde{W}(Z_0)}. \quad 21$$

²¹This result does not depend on the specifications made for the tribute function, (2), or for utility and the

Thus, for cooperative levels of military spending below the opportunistic equilibrium level, the temptation is increasing in Z_0 , the current realization of the endowment.

In contrast to the temptation, the expected gain is independent of the current realization of the endowment. The expected gain from maintained cooperation, denoted by $G(Z - \tilde{W}(Z))$, is found by using (10) and (17):

$$(18) \quad G(Z - \tilde{W}(Z), F(Z)) \\ = \sum_{t=2,4}^{\infty} \int_{Z_1}^{Z_h} \beta^t (1 + \alpha\beta) \\ \times \ln \frac{Z - \tilde{W}(Z)}{k} dF(Z).$$

Again, for $\tilde{W}(Z_t) = \bar{W}(\bar{W}^*, Z_t)$, the gain equals zero, and for $\tilde{W}(Z_t) < \bar{W}(\bar{W}^*, Z_t)$, the gain is strictly positive. Furthermore,

$$\frac{\partial G(Z - \tilde{W}(Z), F(Z))}{\partial \tilde{W}(Z)} \\ = - \sum_{t=2,4}^{\infty} \int_{Z_1}^{Z_h} \beta^t (1 + \alpha\beta) \\ \times \frac{1}{Z - \tilde{W}(Z)} dF(Z) \\ < 0.$$

peaceful investment technology, (5). Specifically, using (1), (3), and (13) with the envelope theorem, one can easily verify the following:

$$- \frac{\partial T(\tilde{W}, Z) / \partial Z}{\partial T(\tilde{W}, Z) / \partial \tilde{W}} \\ = - \frac{u'[Z - i - \bar{W}] - u'[Z - i - \tilde{W}]}{-\beta \Gamma'(\bar{W}, \tilde{W}^*) u'[f(i) + \Gamma(\bar{W}, \tilde{W}^*)] + u'[Z - i - \tilde{W}]}$$

By virtue of the first-order condition, from the maximization of (1) subject to (3) with respect to W , the right-hand side of the above expression simplifies to 1.

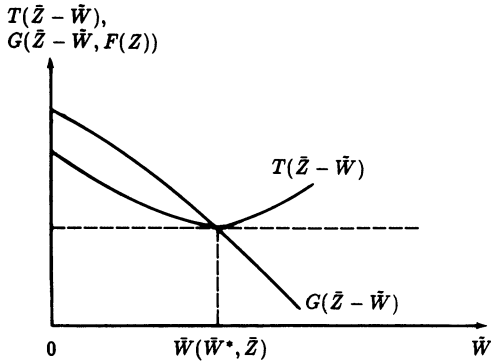


FIGURE 1

Hence, the expected gain from maintained cooperation is smaller if the amount of cooperative military expenditures at all realizations of Z is larger.

The subgame perfection constraint (13) requires that regardless of the value taken on by Z_0 , the fixed, expected gain from maintained cooperation, (18), exceed the temptation to deviate from $\tilde{W}(Z_0)$, given by the difference between (15) and (17). The necessary and sufficient condition for threats and punishments to serve as a perfect substitute for the ability to make binding commitments is that for all possible realizations of Z_0 , $T(Z_0 - 0) - G(Z - 0, F(Z)) \leq 0$.

In the special case that $Z_t = \bar{Z}$ for $t = 0, 2, \dots, \infty$, threats and punishments support the disarmament solution provided that the discount rate is not too large. In this fixed, nonstochastic environment, the cooperative policy, \tilde{W} is independent of time. For this special case, the relationship between the temptation and the gain, as functions of \tilde{W} , is drawn in Figure 1. As shown in the figure, the gain equals the temptation at $\tilde{W} = (\bar{W}(\bar{W}^*, \bar{Z}))$. Moreover, the slope of $G((\bar{Z} - \tilde{W}), \bar{Z})$ is strictly greater in magnitude than that of $T(\bar{Z} - \tilde{W})$ for $0 \leq \tilde{W} < \bar{W}(\bar{W}^*)$ provided that $\beta \geq (1 - \alpha)/(1 + \beta)$.²² At $\tilde{W} = 0$, then, the gain from maintained cooperation exceeds the temptation for op-

portunistic behavior. Since the disarmament policy satisfies the subgame perfection constraint (13), it is the cooperative equilibrium. In other words, the folk theorem of game theory holds in the nonstochastic version of the model. If individuals do not discount the utility of future consumption too heavily, the disarmament outcome can emerge in the equilibrium of an infinite horizon game with threats and punishments (or equivalently, in the cooperative equilibrium).

More generally, however, the disarmament solution is not necessarily supported in a cooperative equilibrium for all values of the endowment. If the distribution of Z is unbounded, then threats and punishments—including punishments more severe than the trigger strategy—cannot support the disarmament outcome. Even if the distribution is bounded, where $[Z_l, Z_h]$ is the support of Z , there is nothing to guarantee that the disarmament policy is consistent with the subgame perfection constraint, (13),—that is, $T(Z_0 - 0) - G(Z - 0, F(Z)) \leq 0$ —for all possible values of Z_0 . Nevertheless, the folk theorem holds in the sense that for a given bounded distribution, there exists a $\beta^* < 1$ such that $\beta \geq \beta^*$ is a sufficient condition for disarmament to be the cooperative equilibrium, supported by a trigger strategy, for all t . In the stochastic environment, the critical value of the discount factor, β^* , depends on the distribution of the endowment, and will generally be larger than $(1 - \alpha)/(1 + \beta)$.²³

B. Military Spending in the Cooperative Equilibrium

When a threat of a reversion to the opportunistic policy is not sufficient to support the efficient outcome for all possible realizations of Z , military spending with such a threat can deter opportunistic behavior. To characterize military spending in the cooperative equilibrium, suppose that $\tilde{W}(Z_t) = 0$ —that is, disarmament—does not satisfy the subgame perfection constraint (13) for $Z_t > \phi$. For values of the endowment in excess of ϕ ,

²²If, for example, $\alpha = 1/2$, a discount factor greater than or equal to 0.367 would satisfy this condition.

²³Note that for more severe punishments, β^* would be smaller.

the optimal contingent cooperative policy, $\tilde{W}(Z_t)$, which specifies the lowest incentive compatible amounts of military spending for each realization of Z_t , satisfies (13) with an equality. Treating (13) as an equality and using the implicit function theorem, one can establish that

$$\frac{\partial \tilde{W}(Z_t)}{\partial Z_t} = - \frac{\partial T(Z_t - \tilde{W}(Z_t)) / \partial Z_t}{\partial T(Z_t - \tilde{W}(Z_t)) / \partial \tilde{W}(Z_t)} = 1.$$

The cooperative policy involves larger amounts of military spending for larger values of the current realization of the endowment. To offset the larger temptation for larger values of Z_t , it is necessary to specify larger \tilde{W} for those possible values of Z_t . In fact, since

$$\frac{\partial T(Z - W(\tilde{Z}))}{\partial Z} = - \frac{\partial T(Z - W(\tilde{Z}))}{\partial W(Z)}$$

(as mentioned above), $\tilde{W}^*(Z_t)$ increases one for one with increases in Z_t in excess of ϕ . Thus, the contingent cooperative policy function, $W(\tilde{Z}_t)$, is flat for $Z_t < \phi$ and kinked at $Z_t = \phi$ with slope equal to 1:

$$(19) \quad \tilde{W}(Z_t) = \begin{cases} 0 & \text{if } Z_t \leq Z_l \leq \phi; \\ Z_t - \phi & \text{if } \phi < Z_t \leq Z_h, \end{cases}$$

where $k \leq \phi < Z_h$. Note that ϕ might be less than Z_l , the lowest possible realization of Z_t . In this case, $\phi \in [k, Z_l)$, and there is positive military spending for all values of the endowment. In any case, military spending by one nation serves to deter the opponent from deviating from the cooperative policy to extract tribute.

According to equation (19), if $\phi = k$, then the cooperative military spending is identical to military spending in the opportunistic equilibrium—that is $\tilde{W}^*(Z_t) = \bar{W}(\bar{W}^*, Z_t)$. However, if $\phi > k$, the level of military spending in the cooperative equilibrium will be less than in the opportunistic equilibrium for all Z . Furthermore, if $\phi > Z_l$, then there are some realizations of the endowment for which the optimal contingent cooperative policy involves disarmament.

Because ϕ is the largest realization of the endowment that would be consistent with disarmament, ϕ is the largest number that satisfies

$$(20) \quad T(\phi - 0) = G(Z - \tilde{W}(Z), F(Z)),$$

where $\tilde{W}(Z)$ is given by equation (19). From equations (15) and (17),

$$\begin{aligned} T(\phi - 0) &= \left[(1 + \beta) \ln \left(\phi + \frac{1 - \alpha}{\alpha} \left(\frac{\alpha}{a} A \right)^{\frac{1}{1 - \alpha}} \right) \right. \\ &\quad \left. \times \frac{1}{1 + \beta} + \beta \ln \beta a \right] \\ &- \left[(1 + \alpha\beta) \ln \frac{\phi}{1 + \alpha\beta} + \beta \ln A + \alpha\beta \ln \alpha\beta \right]. \end{aligned}$$

And, from equations (18) and (19),

$$G(Z - \tilde{W}(Z), F(Z))$$

$$\begin{aligned} &\text{If } \left\{ \begin{array}{l} \frac{\beta^2}{1 - \beta^2} (1 + \alpha\beta) \left[\int_{Z_l}^{\phi} \ln \frac{Z_t}{k} F(Z) \right. \\ \left. + (1 - F(\phi)) \ln \frac{\phi}{k} \right] \\ \text{if } Z_l \leq \phi \leq Z_h; \\ \frac{\beta^2}{1 - \beta^2} (1 + \alpha\beta) \ln \frac{\phi}{k} \\ \text{if } k \leq \phi < Z_l. \end{array} \right. \end{aligned}$$

The value of ϕ that satisfies (20) will be the constant difference between the endowment and the level of cooperative military spending when military spending is positive—that is, according to (19), $\phi = Z_t - \tilde{W}(Z_t)$ for $Z_t > \phi$.

Assume that the necessary and sufficient condition for disarmament to prevail for all t , given by $T(Z_h - 0) \leq G(Z - 0, F(Z))$ (or equivalently, $\phi = Z_h$), is not met. Then the value of ϕ that satisfies (20) is an element of $[k, Z_h)$. It can be shown that if the rate of

time preference satisfies the condition $\beta \geq (1 - \alpha)/(1 + \beta)$, then the value of ϕ that satisfies (20) will be greater than Z_l . To see that $\beta \geq (1 - \alpha)/(1 + \beta)$ implies that $\phi > Z_l$, observe that if $Z - \tilde{W}(Z)$ equals k for all Z , then $T(\phi - 0) = G(Z - \tilde{W}(Z), F(Z))$. Provided that $\beta \geq (1 - \alpha)/(1 + \beta)$, the difference $T(\phi - 0) - G(Z - \tilde{W}(Z), F(Z))$ decreases with increases in the constant $Z - \tilde{W}(Z)$ for $Z > \phi$, over the interval $[k, Z_l]$.²⁴ Hence, if the rate of time preference is not too large, there will be some values of the endowment for which threats and punishments can support the disarmament solution.

Maintaining the assumption that $\beta \geq (1 - \alpha)/(1 + \beta)$, so that $\phi > Z_l$, it is possible to show that the larger is the variance of the endowment, σ^2 , for a given expected value of the endowment, \bar{Z} , the smaller is ϕ —that is,

$$\frac{\partial \phi}{\partial \sigma^2} = - \frac{\frac{\partial T(\phi - 0)/\partial \sigma^2 - \partial G(Z - \tilde{W}(Z), F(Z))/\partial \sigma^2}{\partial T(\phi - 0)/\partial \phi - \partial G(Z - \tilde{W}(Z))/\partial \phi}}{< 0.}$$

Because the greatest number that satisfies (21), ϕ , is less than Z_h —that is, because $T(Z_h - 0) - G(Z - 0, F(Z)) > 0$ —the denominator of the above expression is positive. To see that the numerator of the above expression is nonnegative, note that $\partial T(\phi - 0)/\partial \sigma^2$ equals zero, since the temptation is independent of the variance of the endowment for any given ϕ . Using theorems of second-order stochastic dominance, it is easy to verify that $\partial G(Z - \tilde{W}(Z), F(Z))/\partial \sigma^2$ is nonpositive.²⁵ The proposition follows from the assumptions that the marginal product of the peaceful investment is diminishing and that the marginal product of war goods is constant, which, in conjunction with

the assumption of diminishing marginal utility, imply that the expected gain from maintained cooperation is increasing in the endowment at a diminishing rate.

Similarly, it is possible to show that a smaller expected value of the endowment, \bar{Z} , implies a smaller ϕ , for a given σ^2 —that is,

$$\frac{\partial \phi}{\partial \bar{Z}} = - \frac{\frac{\partial T(\phi - 0)/\partial \bar{Z} - \partial G(Z - \tilde{W}(Z), F(Z))/\partial \bar{Z}}{\partial T(\phi - 0)/\partial \phi - \partial G(Z - \tilde{W}(Z))/\partial \phi}}{> 0.}$$

Again, the denominator of the above expression is positive. Notice that the temptation is independent of the expected value of the endowment for any given ϕ . Hence, $\partial T(\phi - 0)/\partial \bar{Z}$ equals zero. Upon inspection of (20), where the expected gain from maintained cooperation is defined for $\phi > Z_l$, it becomes obvious that $\partial G(Z - \tilde{W}(Z), F(Z))/\partial \bar{Z}$ is positive. The intuition here is as follows. Holding ϕ fixed and increasing the expected value of the endowment means that conditional on $Z < \phi$, when there is disarmament and the two-period expected utility from cooperation depends on Z , the two-period expected utility from cooperation is higher. And, conditional on $Z > \phi$, the two-period expected utility from cooperation is unchanged. Since the utility obtained in the punishment is independent of Z , the expected gain from maintained cooperation increases with increases in \bar{Z} . Hence, the numerator is negative and the whole expression is positive.

From equation (19), a smaller ϕ implies that for a given realization of Z_l , the level of cooperative military spending is higher.²⁶ Therefore, if the cooperative equilibrium is empirically relevant, assuming that the cooperative equilibrium threshold level of the endowment, ϕ , is greater than the lowest possible realization of the endowment, Z_l , one would find, for a given realization of the endowment, that the associated level of mili-

²⁴See Figure 2 where the difference $T(\phi - 0) - G(Z - \tilde{W}(Z), F(Z))$ is drawn as a function of $Z - \tilde{W}(Z)$ for $Z > \phi$, assuming that $\beta \geq (1 - \alpha)/(1 + \beta)$.

²⁵See the Appendix for a proof.

²⁶Also, see Figure 3.

tary spending is larger, the larger is the variance of the endowment and the smaller is the mean value of the endowment. Alternatively, if the opportunistic equilibrium were empirically relevant, one would find no relationship between a nation's military spending and the variability or the mean value of the endowment, for a given realization of the endowment.

It is important to note that while these empirical implications do not depend on the specifications made for utility and the peaceful investment technology, (5), they do depend on two assumptions. First, these implications require that the threshold level of the endowment is greater than the lowest possible realization of the endowment, such that there are some values of the endowment for which disarmament prevails. The assumption is important for the proofs sketched above. If instead the equilibrium cooperative policy always involves positive military spending, then the expected gain will be independent of the distribution of the endowment. Even so, it is still possible to make some positive predictions. Specifically, although the hypothetical changes in the distribution of Z would not have an impact on the difference $T(\cdot) - G(\cdot)$ over the range $[k, Z_l]$, these changes would alter the difference in the interval $(Z_l, Z_h]$. An increase in \bar{Z} or a decrease in σ^2 would push $T(\cdot) - G(\cdot)$ down over the interval $(Z_l, Z_h]$. Since ϕ is the largest number that makes $T(\cdot)$ equal to $G(\cdot)$, if the effect is large enough to result in an intersection of the difference $T(\cdot) - G(\cdot)$ and the zero horizontal axis (see Figure 2) over the interval $(Z_l, Z_h]$, then, assuming that the original ϕ is less than Z_l , the change in the distribution would imply a discontinuous jump in $\phi \in [Z_l, Z_h]$, or equivalently, a drop in cooperative military spending for each realization of the endowment.

Second, and more importantly, these implications depend on the linear specification of the tribute function. If instead the marginal product of war goods were diminishing, then the expected gain could be independent of the endowment or decreasing in the endowment at an increasing rate. As a consequence, exercises involving hypothetical changes in the distribution might not

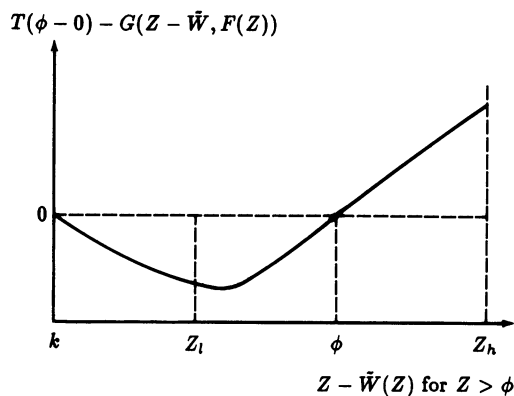


FIGURE 2

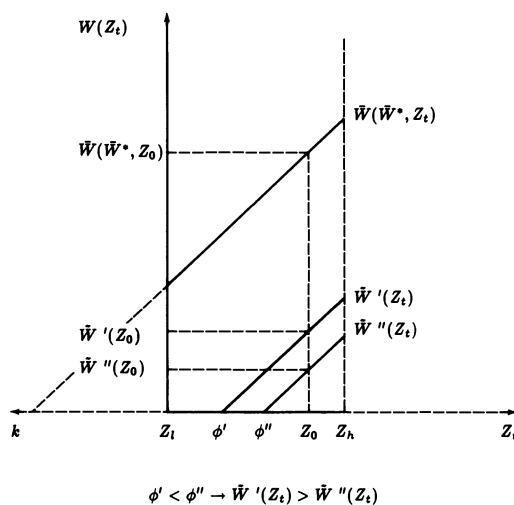


FIGURE 3

yield implications that would enable one to distinguish empirically the opportunistic equilibrium from the cooperative equilibrium.²⁷

Generally, in a stochastic environment, threats and punishments do not necessarily serve as a perfect substitute for the ability to make commitments. Although the punishment strategy employed in the analysis does

²⁷ Deriving and implementing more general testable empirical implications of the model are left for future research.

not involve the most severe punishment, consideration of more severe punishments will not qualitatively alter the discussion above. Specifically, the most severe punishment can only yield a fixed expected gain to maintained cooperation. The lowest utility that a government could expect to obtain if it followed its minimax strategy and the opponent devoted all of its resources to military spending starting in period $t = 2$ establishes an upper bound to the gain from maintained cooperation that could be expected by the governments with the most severe punishment strategy. There is nothing to guarantee that the fixed, expected gain will dominate the temptation, for all realizations of the endowment, where the cooperative policy is the disarmament policy. While strategic threats and punishments would be more likely to support the efficient solution if the discount rate is low, a low discount rate is not always sufficient to ensure that the efficient solution emerges in a cooperative equilibrium. Hence, while threats and punishments can reduce the magnitude of the resources wasted away in military spending, they cannot necessarily eliminate the time-inconsistency problem of the commitment to peaceful policies.

III. Conclusion

This paper has investigated the role of strategic considerations in an economic model of international conflict. The analysis has shown that when there is repeated interaction between nations, threats and punishments, which at least serve as a partial substitute for a government's ability to make commitments, might be sufficient for sustaining the *ex ante* optimal, but otherwise time-inconsistent, disarmament policy. Threats and punishments are more likely to support the commitment policy, as the discount factor approaches one.

If the fixed expected gain from maintained cooperation does not exceed the temptation to deviate from the peaceful policy for all possible realizations of the endowment, then the model predicts that there will be some periods of positive production of armaments. Because the temptation to deviate from the cooperative policy is increasing in

the endowment, for values of the endowment that fall below an endogenously determined threshold value, the contingent cooperative policy involves only peaceful policies, whereas for values of the endowment that exceed the threshold value, the cooperative policy involves positive military expenditures, the magnitude of which is dependent on the current realization of the endowment.²⁸ Hence, fluctuations in military spending are an endogenous result of fluctuations in the endowment in both the opportunistic and the cooperative equilibria. This theoretical result casts doubt on the validity of empirical work that treats military expenditures as exogenous.²⁹

Although the relationship between military spending and the endowment is positive regardless of whether governments are acting cooperatively or opportunistically, there are qualitative differences in this relationship depending on whether governments are acting cooperatively or opportunistically. In particular, if governments are acting cooperatively, the larger is the variance of the endowment and smaller is the mean value of the endowment, the smaller is the equilibrium threshold level of the endowment. Consequently, for a given realization of the endowment, a nation's military spending is positively related to the variability of the endowment and negatively related to the mean value of the endowment. In contrast, if nations are acting opportunistically, their military spending would not be related to the variance of the endowment or the mean value of the endow-

²⁸Although the notion of positive military expenditures in a cooperative equilibrium might seem counterintuitive, this result is very similar to the results of Grossman and Van Huyck (1988), who study the role of reputation in the problem of sovereign debt, interpreted as a contingent claim. Positive military expenditures on armaments in the cooperative equilibrium would correspond to their notion of excusable default, which is to be clearly distinguished from repudiation or a reversion to the opportunistic policy.

²⁹See for example Hall (1986), who assumes that military spending is exogenous to identify the role of consumption in macroeconomic fluctuations. As discussed above—fn. 1—the existing empirical evidence concerning the (statistical) causal relation between military spending and other aggregate economic variables is ambiguous.

ment, for a given realization of the endowment.

Finally, it is important to observe that in the cooperative equilibrium, military spending is not directly driven by the possibility of extracting resources from an enemy nation. Rather, military spending serves to deter the nations from building armaments in an opportunistic effort to obtain tribute.

APPENDIX

To see that a larger variance of the endowment implies an equal or smaller expected gain from maintained cooperation, consider two distributions, $F_1(Z)$ and $F_2(Z)$, that have identical supports and mean values:

$$\int_{Z_l}^{Z_h} Z [dF_1(Z) - dF_2(Z)] = 0,$$

or equivalently,

$$(A1) \quad \int_{Z_l}^{Z_h} [F_1(Z) - F_2(Z)] dZ = 0.$$

If $dF_1(Z)$ and $dF_2(Z)$ satisfy the single crossing property, there exists a $Z'\epsilon(Z_l, Z_h)$ such that

$$(A2) \quad F_1(Z) - F_2(Z) \begin{cases} \leq 0 & \text{if } Z' \leq Z \leq Z_h; \\ \geq 0 & \text{if } Z_l \leq Z \leq Z'. \end{cases}$$

The two distributions that satisfy (A1) and (A2) also satisfy the following:

$$(A3) \quad \int_{Z_l}^{Z'} [F_1(Z) - F_2(Z)] dZ \geq 0,$$

for all $y \epsilon [Z_l, Z_h]$. The expressions in (A1) and (A3) mean that $F_2(Z)$ dominates $F_1(Z)$ in the sense of second-order stochastic dominance. $F_1(Z)$, which is obtained from a mean preserving spread of $F_2(Z)$, has a larger variance than $F_2(Z)$. (See Michael Rothschild and Joseph Stiglitz, 1970).

Using (20) for $\phi \geq Z_l$, a switch from $F_2(Z)$ to $F_1(Z)$, (an increase in the variance) results in the following change in the expected gain from maintained cooperation:

$$(A4) \quad \int_{Z_l}^{\phi} \ln Z [dF_1(Z) - dF_2(Z)] \\ + [F_2(\phi) - F_1(\phi)] \ln \phi.$$

Integrating by parts, (A4) becomes

$$(A5) \quad \ln Z [F_1(Z) - F_2(Z)] \Big|_{Z_l}^{\phi} \\ - \int_{Z_l}^{\phi} \frac{1}{Z} [F_1(Z) - F_2(Z)] dZ \\ + [F_2(\phi) - F_1(\phi)] \ln \phi,$$

which simplifies to the following:

$$(A6) \quad - \int_{Z_l}^{\phi} \frac{1}{Z} [F_1(Z) - F_2(Z)] dZ,$$

since $F_1(Z_l) = F_2(Z_l) = 0$. Integrating (A6) by parts gives the following expression for the change in the expected gain due to the mean preserving spread of the distribution:

$$(A7) \quad - \frac{1}{\phi} [\hat{F}_1(\phi) - \hat{F}_2(\phi)] \\ + \int_{Z_l}^{\phi} - \frac{1}{\phi^2} [\hat{F}_1(Z) - \hat{F}_2(Z)],$$

where $\hat{F}(Z) \equiv \int_{Z_l}^Z F(Z) dZ$ and $\hat{F}(Z_l) = 0$. By virtue of (A3), the expression above is nonpositive. Hence, an increase in the variance of the endowment implies an equal or smaller expected gain from maintained cooperation.

REFERENCES

- Abreu, Dilip**, "External Equilibria of Oligopolistic Supergames," *Journal of Economic Theory*, June 1986, 39, 191-225.
- Axelrod, Robert**, *The Evolution of Cooperation*, New York: Basic Books, 1984.
- Barro, Robert and Gordon, David**, "Rules, Discretion and Reputation in a Model of Monetary Policy," *Journal of Monetary Economics*, July 1983, 12, 101-21.
- Brams, Steven and Kilgour, Marc**, "The Path to Stable Deterrence," in U. Luterbacher and M. Ward, Boulder, eds., *Dynamic Models of International Conflict*, Boulder, CO: Lynn Rienner Publishers, 1985, 11-25.
- _____ and _____, "Deterrence Versus Defense: A Game-Theoretic Model of Star Wars," C. V. Starr Center for Applied Economics, Research Report No. 86-35, 1986.
- Brams, Steven and Wittman, Donald**, "Non-myopic Equilibria in 2x2 Games," *Conflict Management and Peace Science*, Fall 1981, 6, 39-62.
- Brito, Dagobert**, "A Dynamic Model of an Armaments Race," *International Economic Review*, June 1972, 13, 359-75.
- _____ and **Intriligator, Michael**, "Conflict, War, and Redistribution," *American Political Science Review*, Winter 1985, 79, 943-57.
- Canzoneri, Matthew and Gray, Jo Anna**, "Mone-

- tary Policy Games and the Consequences of Non-Cooperative Behavior," *International Economic Review*, October 1985, 26, 547-64.
- _____ and Henderson, Dale, "Strategic Aspects of Macroeconomic Policymaking in Interdependent Economies: Reputation in Repeated Games," mimeo., 1986.
- _____ and _____, "Is Sovereign Policy-making Bad?" *Carnegie-Rochester Conference Series on Public Policy*, Spring 1988, 28, 93-140.
- Feinman, Joshua, Garber, Peter, and Garfinkel, Michelle, "Financial Warfare: Attacks on Credit Mechanisms," mimeo., 1986.
- Friedman, James, "A Non-cooperative Equilibrium for Supergames," *Review of Economic Studies*, January 1971, 38, 1-12.
- Garcia-Mila, Teresa, "Government Purchases and Real Output: An Empirical Analysis and Equilibrium Model with Public Capital," mimeo., 1987.
- Grossman, Herschel and Van Huyck, John, "Seigniorage, Inflation, and Reputation," *Journal of Monetary Economics*, July 1986, 18, 21-34.
- _____ and _____, "Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation, and Reputation," *American Economic Review*, December 1988, 78, 1088-97.
- Hall, Robert, "The Role of Consumption in Economic Fluctuations," in *The American Business Cycle*, Robert Gordon, ed., Chicago: University of Chicago Press, 1986, 237-55.
- Intriligator, Michael, "Strategic Considerations in the Richardson Model of Arms Races," *Journal of Political Economy*, April 1975, 83, 339-53.
- _____, "Research on Conflict Theory: Analytic Approaches and Areas of Application," *Journal of Conflict Resolution*, June 1982, 26, 307-27.
- _____ and Brito, Dagobert, "Can Arms Races Lead to the Outbreak War?," *Journal of Conflict Resolution*, March 1984 28, 63-84.
- Maoz, Zeev, "Decision-Theoretic and Game-Theoretic Models on International Conflict," in *Dynamic Models of International Conflict*, U. Luterbacher and M. Ward, eds., Boulder, CO: Lynn Rienner Publishers, 1985, 76-11.
- Milward, Alan, *The New Order and the French Economy*, Oxford: Clarendon Press, 1970.
- North, Douglas, *Structure and Change in Economic History*, New York: W. W. Norton & Co., 1981.
- Olson, Mancur, Jr. and Zeckhauser, Richard, "An Economic Model of Alliances," *Review of Economics and Statistics*, August 1966, 48, 266-79.
- Riddell, Tom, "U.S. Military Power, the Terms of Trade, and the Profit Rate," *American Economic Review Papers and Proceedings*, May 1988, 78, 60-65.
- Robbins, Lionel, *The Economic Causes of War*, New York: Macmillan, 1940.
- Rothschild, Michael and Stiglitz, Joseph, "Increasing Risk I: A Definition," *Journal of Economic Theory*, September 1970, 2, 225-43.
- Schelling, Thomas, *The Strategy of Conflict*, Cambridge, MA: Harvard University Press, 1960.
- Shepherd, William, "Self Interest and National Security," *American Economic Review Papers and Proceedings*, May 1988, 78, 50-54.
- Shubik, Martin, "Game Theory Models of Strategic Behavior and Nuclear Deterrence," Cowles Foundation Discussion Paper No. 829, 1987.
- Smith, R. P., "Alternative Models of Military Expenditure," Birbeck College Discussion Paper in Economics, No. 87-17, 1987.
- van der Ploeg, F. and de Zeeuw, A. J., (1986a) "Conflict Over Arms Accumulation in Market and Command Economies," mimeo., 1986.
- _____ and _____, (1986b) "Perfect Equilibrium in a Model of Competitive Arms Accumulation," mimeo., 1986.
- Zagare, Frank, "The Pathologies of Unilateral Deterrence," in *Dynamic Models of International Relations*, U. Luterbacher and M. Ward, eds., Boulder, CO: Lynn Rienner Publishers, 1985, 52-75.