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Stable alliance formation in distributional conflict

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Abstract

This paper develops a positive analysis of alliance formation, building on a simple economic model that features a "winner-take-all" contest for control of some resource. When an alliance forms, members pool their efforts in that contest and, if successful, apply the resource to a joint production process. Due to the familiar free-rider problem, the formation of alliances tends to reduce the severity of the conflict over the contestable resource. Despite the conflict that arises among the winning alliance's members over the distribution of their joint product, under reasonable conditions, this effect alone is sufficient to support stable alliance formation in a noncooperative equilibrium.

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1. Introduction

Conventional economic analyses of the allocation of resources among various productive uses and the distribution of the product generated from those uses take as given the existence of well-defined and costlessly enforced property rights. The emerging literature on conflict and predation, however, shows that allowing for the possibility of conflict in economic interactions can have profound implications for the distribution of resources.[1]

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[1] See, for example, Hirshleifer (1991, 1995), Skaperdas (1992) and Grossman and Kim (1995). Garfinkel and Skaperdas (2000) provide a brief overview.

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But, that literature seems somewhat incomplete in that it abstracts from the possibility that alliances form.[2]

This paper develops a positive analysis of alliance formation. The analysis is based on a simple economic model that features a "winner-take-all" contest for control of some resource—for example, territory. Without the formation of alliances, each individual exerts some effort to secure the resource, which in turn is applied to the production of a homogeneous consumption good. By contrast, when an alliance forms, members pool their efforts in that contest. If successful, the members in turn apply the resource to a joint production process.

Moving beyond the traditional theory of alliances that follows the pioneering work of Olson and Zeckhauser (1966),[3] the present analysis does not take the membership of alliances as given. Nor is there any presumption that peace prevails among members of an alliance. Rather, the distribution of the output from joint production is subject to another, separate conflict—that is, between the members of the alliance. The analysis shows that, just as the possible emergence of conflict between individuals in their economic interactions can have important implications for the equilibrium distribution of resources and income, the possibility of conflict between individuals within an alliance should not be ignored.[4]

Nevertheless, borrowing from this traditional theory, one might naturally look to the public-good nature of defense and appropriative efforts to explain the emergence of alliances—for example, the cost-saving advantages realized when neighboring nations take defensive measures against a common enemy (Sandler, 1999).[5] Skaperdas (1998) and Noh (2002) show in different though related settings that a conflict technology having this sort of property is, in fact, critical for the emergence of an alliance.

The analysis of the present paper suggests, however, that such a technology might not even be necessary. A potential benefit of the formation of alliances, captured in neither Skaperdas (1998) nor Noh (2002), derives from the presence of the free-rider problem. Specifically, the incentive for each member of the alliance to contribute to the group's collective effort in securing the contestable resource is reduced by the alliance's size, since the benefit possibly realized would have to be shared with the other members, whereas the

[2] Two notable exceptions—Skaperdas (1998) and Noh (2002)—are discussed below.

[3] For a survey of this literature, including applied work, see Sandler and Hartley (2001).

[4] The literature on collective rent seeking—see, for example, Nitzan (1991)—similarly considers settings in which there are two levels of conflict: that which emerges between groups and that which emerges within a group. However, most of the analyses in this literature effectively treat the two levels of conflict as one. For given each group's pre-determined sharing rule, each member's contribution to his respective group's effort in the inter-group conflict jointly determines the outcome of both that conflict and the intra-group conflict. By contrast, the present analysis treats the resolution of the conflict within the alliance as distinct from that between alliances, and more importantly assumes no mechanism by which members of an alliance can commit to a sharing rule. (Katz and Tokatlidu (1996) and Wärneryd (1998) have a similar structure).

[5] Also see Sandler (1993) who surveys the previous literature on the public-good nature of an alliance's defense. Alesina and Spolaore (2000) recently consider the importance of international conflict in the equilibrium determination of the size and number of the nation-states, suggesting in their concluding remarks that the nation-state might be interpreted as an alliance. However, like the traditional theory of alliances, they presume that peace prevails among members of the nation-state.

cost is borne privately by the individual member.[6] Hence, the conflict directly over the contestable resource is reduced. Of course, a new conflict arises with the formation of alliances, and as already suggested, additional resources must be expended by each member of the winning alliance to secure her share of the alliance's product. Nonetheless, relative to the case of individual conflict, the formation of alliances tends to reduce the overall severity of conflict.[7]

An application of the theory of endogenous coalition structures—in the spirit of, for example, Bloch (1996), Chwe (1994), Ray and Vohra (1997, 1999), and Yi (1997)—shows that this negative net effect on the severity of conflict alone generally is sufficient to support the formation of alliances in a noncooperative equilibrium. Now, when the total number of individuals is very small, there is virtually no room to diffuse the inter-alliance conflict, whereby the structure can be made incentive compatible. However, when the number of individuals involved is sufficiently large, there exists at least one stable multi-member alliance structure, and beyond that multiple configurations are possible. In such cases, while the expected gains under alliance formation summed across all individuals might be larger when alliances are not of the same size as has been suggested by Katz and Tokatlidu (1996), they need not be. The overriding determinant, given the total number of individuals involved, is instead the number of alliances.[8] Specifically, the analysis finds that the expected gains from alliance formation in the aggregate relative to individual conflict are greater when the number of alliances is larger and the alliances are smaller in size.[9]

In what follows, the next section presents the model of conflict which allows for the formation of multi-member alliances. Section 3 establishes the benchmark case of individual conflict. Then, treating the pre-conflict determination of the structure of alliances as given, Section 4 characterizes the allocation of resources and payoffs generally and, in the case of a symmetric alliance structure, illustrates the benefit of alliance formation to reduce the severity of conflict. Section 5 then studies the endogenous formation of alliances, characterizing stable alliance structures and their welfare implications. Finally, Section 6 offers some concluding remarks, including a brief discussion of possible extensions of the analysis.

[6] In Skaperdas (1998), the outcome of each conflict is determined by the parties' relative strategic endowments, which are given exogenously. In Noh (2002), the free-rider problem does not arise, simply because the cost of each member's contribution to the alliance's collective effort is borne by the entire alliance. For that part of the individual's inalienable endowment which is not used in the collective effort to secure the contestable resource is used in joint production. The alliance's output, in turn, is distributed equally to the members—i.e., according to the ex ante optimally chosen sharing rule.

[7] Wärneryd (1998) first suggested such an effect in his explanation of the emergence of a federalist structure of jurisdictional interaction. The analysis of the present paper goes beyond this novel idea towards the endogenous determination of the number and size of the alliances in conflict.

[8] To be sure, Katz and Tokatlidu (1996) take the number of groups (2) as given, and study the implications of asymmetric group size.

[9] By contrast, Baik and Lee (2001), who similarly study the endogenous determination of the number and sizes of groups, tend to predict the emergence of just one alliance. As discussed below, the difference in these predictions can be attributed, in large part, to the fact that, in their analysis, like that of Noh (2002), the "prize" from the inter-alliance contest is distributed to alliance members according to a *pre-committed* rule. While clouding the distinction between the inter-alliance conflict and the intra-alliance conflict, this distribution mechanism presumes an element of cooperation within the alliance.

2. Analytical framework

Consider an environment populated by N identical, risk-neutral individuals, $\mathscr{I} = \{1, 2, \ldots, N\}$, who participate in a three-stage game. In the first stage, agents $i \in \mathscr{I}$ form alliances. An alliance is defined as any subset of the population, $\mathscr{A}_k \subseteq \mathscr{I}$, with membership $n_k \geq 1$, where $k = 1,2,\ldots, A$ and A denotes the total number of alliances. For future reference, let the alliance structure be indicated by $S = \{n_1, n_2, \ldots, n_A\}$, with the alliances ordered such that $n_1 \geq n_2 \geq n_3 \ldots \geq n_A$. By definition, all individuals belong to an alliance. However, an alliance need not include more than one member. Moreover, this framework admits the possibility that everyone comes together to form a single alliance—the *grand alliance*: $\mathscr{A}_1 = \{1,2,\ldots,N\}$.

2.1. Stage 2: conflict between alliances

In the second stage, all individuals participate in a winner-take-all conflict/contest over a resource X, which is necessary for the production of a homogeneous consumption good in the third stage. They participate either collectively with others or alone, as dictated by the alliance structure determined in stage one. For any given configuration of alliances, each member $i \in \mathscr{A}_k$ chooses how much she will contribute to the alliance's appropriative effort, m_i.[10] The probability that alliance k wins the conflict and successfully secures the entire resource X is determined by

$$\mu_k = \frac{\sum\limits_{i \in A_k} m_i}{\sum\limits_{j=1}^{A} \sum\limits_{i \in \mathscr{A}_j} m_i} \tag{1}$$

if $\sum_{j=1}^{A} \sum_{i \in \mathscr{A}_j} m_i > 0$; otherwise, $\mu_k = 1/A$ for all k.[11]

By assumption here, members of any alliance k with $n_k > 1$ have no special advantage over those individuals who choose to participate in the conflict on their own.[12] Nevertheless, this formulation captures one aspect of the public-good nature of defense

[10] Since production is not possible until the resource is secured, the cost of this effort, as specified below, can be interpreted as foregone leisure.

[11] This specification, first introduced by Tullock (1980) for individual rent seeking, is the contest success function most commonly used in the conflict/contest, rent-seeking literature. As argued below, although it admits the possibility of a corner solution for all members of all alliances, such a solution is not a possible equilibrium outcome. See Hirshleifer (1989) who discusses the properties of this specification and related ones. Note that, under the maintained assumption of risk-neutrality, an alliance's probability of winning and taking the entire prize X, μ_k, may be interpreted alternatively as its resource share.

[12] To allow for such effects, Skaperdas (1998) modifies Eq. (1) as follows:

$$\mu_k = \frac{\left(\sum_{i \in \mathscr{A}_k} m_i\right)^{\gamma}}{\sum_j^A \left(\sum_{i \in \mathscr{A}_j} m_i\right)^{\gamma}}.$$

With $N = 3$, he finds that a stable alliance between two of the three agents is possible only when $\gamma > 1$ (i.e., under super-additivity). Noh (2002) obtains a similar result assuming a slightly different specification to capture the advantage that multi-member alliances have in conflict.

spending. In particular, appropriative efforts by different members of a given alliance are perfect substitutes for one another. Regardless of who provides any additional effort, all members enjoy the increased probability of securing the resource X it implies.

2.2. Stage 3: joint production and conflict within the alliance

To fix ideas, suppose that alliance k, with $n_k>1$, successfully captures the resource X. Individuals not belonging to that alliance, $i\in\mathscr{A}_{k'}$ where $k'\neq k$, receive nothing, implying that their second-stage efforts result only in a loss, $u_{ik'}=-m_i$, over the three stages.

Turning to the members i of the winning alliance k, each $i\in\mathscr{A}_k$ is identically endowed with a unit of labor, which she allocates to productive activities, l_i, and appropriative or security related activities, s_i, subject to

$$1 = l_i + s_i. \tag{2}$$

These activities along with X, in turn, deliver goods for consumption at the end of the stage. Specifically, individuals $i\in\mathscr{A}_k$ collectively combine the resource X with a fraction of their labor endowment, $l_i=1-s_i$ in a joint (linear) production process to yield a homogeneous consumption good. Generally, for $n_k\geq 1$ using Eq. (2), the alliance's total product, Y_k, is specified as

$$Y_k = \sum_{i\in\mathscr{A}_k}[1-s_i]X/n_k. \tag{3}$$

Although X might be considered a public good from the perspective of the second-stage (winner-take-all) conflict, at this stage, X would be interpreted as a purely private good.[13]

For $n_k>1$, each member must also devote a strictly positive effort, $s_i>0$, towards securing a share of the final product. This latter activity, reflecting the conflict that emerges within the winning alliance, detracts from production. Assume that the *share* of final output, Y_k, enjoyed by agent $i\in\mathscr{A}_k$, σ_{ik}, depends on her security effort s_i, distinct from m_i, and on the effort by everyone else in her alliance, s_j for $j\neq i\in\mathscr{A}_k$. More formally, for $n_k\geq 1$,

$$\sigma_{ik} = \frac{s_i}{\sum_{j\in\mathscr{A}_k} s_j} \tag{4}$$

if $\sum_{j\in\mathscr{A}_k} s_j > 0$; otherwise $\sigma_{ik}=1/n_k$ for all $i\in\mathscr{A}_k$. Each member $i\in\mathscr{A}_k$, then, obtains a payoff given by $u_{ik}=\sigma_{ik}Y_k-m_i$. Whether her alliance secures the resource or not, each

[13] Specifying production as a joint process is common in the economics literature on conflict. (See, for example, the survey by Garfinkel and Skaperdas, 2000). Allowing for complementarities or increasing returns in production would provide another potential benefit of group formation. However, given the linear homogeneity of the technology as specified here, one need not suppose that there is any sort of interaction between alliance members in production. An alternative interpretation of the production technology (Eq. (3)) is that each member of the winning alliance takes an equal share of X at the beginning of the third stage and produces in isolation of the others. In this case, the share σ_{ik}, defined below in Eq. (4), would represent the fraction of her own product that member i defends and that which she captures from the other members of her alliance k.

individual alone bears the cost of the contribution she makes to her alliance's effort in the second-stage conflict. Even if there were no conflict within the winning alliance so that $s_i = 0$ for all $i \in \mathscr{A}_k$ and $\sigma_{ik} = 1/n_k$, the free-rider problem would be relevant.

However, the analysis does not assume that "peace" prevails among the alliance members. That is, while involved in a joint production process in the third stage, individuals must worry about the eventual distribution of the alliance's product. Following Grossman (2001), one can think of the members' security or guarding efforts which determine this distribution, s_i, as a process by which effective property rights are created and output jointly produced is shared. That some additional resources are required by each individual to secure a share of the final output makes alliances less appealing. Indeed, in supposing that some effort is required in the production process and in securing a share of the final output, the analysis of the third stage of this model captures the fundamental trade-off between production and appropriation highlighted by Haavelmo (1954, pp. 91–98) and considered more recently by, for example, Hirshleifer (1989), Garfinkel (1990), Skaperdas (1992), and Grossman and Kim (1995).[14]

3. Individual conflict

Before moving on to the analysis of the equilibrium allocation of resources given the alliance structure, consider the case of *individual conflict*. Specifically, suppose that each alliance has only one member: $A = N$ and $S = \bar{S} \equiv \{1, \ldots, 1\}$. In this case with conflict over X only, each individual i (or alliance k) chooses m_i to maximize her expected payoff, equal to her expected consumption in the third stage net of the cost of her effort in securing the contestable resource in the second stage, or $u_i^{\mathrm{e}} = \mu_i X - m_i$, subject to Eq. (1).[15] Assume everyone makes this choice simultaneously.

The specification for the contest success function (Eq. (1)) generally implies that, if no appropriative effort were made by anyone, then any individual could capture the contestable resource with near certainty by putting forth an infinitesimally small amount of effort. Since no one would leave such an opportunity unexploited, the "peaceful" outcome where $m_i = 0$ for all $i \in \mathscr{I}$ cannot be an equilibrium. Thus, each

[14] That such a trade-off does not emerge in the second stage might appear to be important for the central results of the analysis. What is important here, however, is that individuals do not fully internalize the benefits of their efforts in fighting over X relative to the costs they incur. In particular, the findings of this analysis would follow if it were alternatively based on a framework that is more in line with the collective rent-seeking literature such as that in the work of Noh (2002), provided that individuals also valued leisure. By the same token, the qualitative results would remain the same if the analysis were based on a model in which there was no production in the third stage, as in the works of Katz and Tokatlidu (1996) and Wärneryd (1998).

[15] This cost is expressed in the equivalent units of expected consumption. Since $n_k = 1$, Eq. (4) implies, independent of the rest of the alliance structure, that $s_i = \bar{s}(1) = 0$ and $\sigma_{ik} = 1$. Then, Eq. (3) implies that $\sigma_{ik} Y_k = X$.

individual's optimizing choice of m_i satisfies the first-order condition, $[M-m_i]X=M^2$, where $M=\sum_{j\in\mathscr{I}} m_j$, implying the unique, symmetric Nash equilibrium:

$$\bar{m}\equiv m(1,\ \bar{S})=(N-1)X/N^2 \tag{5a}$$

$$\bar{u}^{\mathrm{e}}\equiv u^{\mathrm{e}}(1,\ \bar{S})=X/N^2>0 \tag{5b}$$

for $i\in\mathscr{I}$. As shown in Eq. (5b), each individual's expected payoff is strictly positive, increasing in the amount of the contested resource, X, but decreasing in the total number of people competing for that resource, N. We will refer back to this outcome later as it provides a benchmark against which the gains of multi-member alliance formation can be measured.

4. The allocation of resources and expected payoffs given the alliance structure

Treating the pre-conflict determination of the alliance structure S as given, the analysis now considers the allocation of resources in the second and third stages. Each individual aims to maximize her expected payoff which equals, as in the case of individual conflict, her expected consumption in the third stage net of the cost of her effort in securing the contestable resource in the second stage expressed in the equivalent units of expected consumption: $u_{ik}^{\mathrm{e}}=\mu_k\sigma_{ik}Y_k-m_i$. In this dynamic setting, the amount of resources available to anyone in the third stage will, of course, depend on second-stage choices. All individuals, when making their second-stage choices, will take this influence into account. In accordance with the equilibrium notion of subgame perfection, then, we solve the model backwards, starting with the third and final stage.

4.1. The outcome of the intra-alliance conflict

Given the outcome of the second-stage conflict over X and m_i, Eqs. (3) and (4) imply that the payoff to each member i of the winning alliance k, $u_{ik}=\sigma_{ik}Y_k-m_i$, can be written as

$$u_{ik}=\frac{s_i}{\sum\limits_{j\in\mathscr{A}_k} s_j}\left[\sum_{j\in\mathscr{A}_k}(1-s_j)\frac{X}{n_k}\right]-m_i. \tag{6}$$

Each individual $i\in\mathscr{A}_k$ chooses s_i to maximize this expression. Assume that alliance members make their third-stage choices simultaneously.

The conflict technology shown in Eq. (4) precludes the "peaceful" outcome where $s_i=0$ for all $i\in\mathscr{A}_k$ and $n_k>1$.[16] As such, the following condition must be satisfied at an optimum:

$$\frac{\sum_{j\neq i\in\mathscr{A}_k} s_j}{\sum_{j\in\mathscr{A}_k} s_j}\left[\sum_{j\in\mathscr{A}_k}(1-s_j)\right]\geq s_i, \tag{7}$$

with strict equality for $s_i<1$. Not surprisingly given the symmetry of the alliance's membership, this condition implies that members choose the same labor allocation, $s_i=s$ and, at the same time, an interior optimum: $s_i\in(0,1)$ for all $i\in\mathscr{A}_k$.[17] Using Eqs. (3), (4) and (6), condition (7) as a strict equality implies the following Nash equilibrium of the third stage:

$$s_i=(n_k-1)/n_k, \tag{8a}$$

$$u_{ik}=X/n_k^2-m_i \tag{8b}$$

for all $i\in\mathscr{A}_k$. In this equilibrium, each member of the winning alliance enjoys an equal share of final output: $\sigma_{ik}=1/n_k$, which is decreasing in the size of the alliance.[18] However, given m_i, because a larger n_k implies a greater dilution of the prize X and a greater diversion of effort away from production towards security, the payoff is decreasing in the square of the size of the alliance.

4.2. The outcome of the inter-alliance conflict

Now consider the second-stage conflict between alliances, again with the alliance structure fixed. Each individual i belonging to alliance k chooses m_i to maximize the *expected* value of Eq. (8b), given by

$$u_{ik}^{\mathrm{e}}=\mu_k X/n_k^2-m_i, \tag{9}$$

subject to the conflict technology, μ_k, as specified in Eq. (1). Individuals in all A alliances make their decisions simultaneously. In Eq. (9), the first term represents the product enjoyed by member i of alliance k, having won the conflict, weighted by the winning probability, μ_k. The second term represents the utility cost of fighting over the contestable

[16] The reasoning here is analogous to that sketched above with respect to the conflict technology shown in Eq. (1) (see Section 3).

[17] Note, in particular, that if $s_i=1$ held for some i, then it would have to hold for all $i\in\mathscr{A}_k$. But then the alliance's total output (Y_k) and thus the left hand side of Eq. (7) would be equal to zero, yielding a self-contradiction.

[18] Of course, as indicated earlier, the specification for the conflict resolution technology (Eq. (4)) implies that for $i\in\mathscr{A}_k$, where $n_k=1$, $s_i=\bar{s}=0$.

resource expressed in equivalent units of expected consumption; it is borne solely by the individual regardless of the outcome of that conflict.

Although the conflict technology as specified in Eq. (1) implies that $\sum_{j=1}^{A}\sum_{i\in\mathscr{A}_j} m_i > 0$, a fully interior solution is not guaranteed for all configurations of alliances when $A>2$. That is to say, the members of one or more alliances might choose $m_i=0$. However, the stability of a given configuration does require that all alliances actively participate in the second-stage conflict.[19] In anticipation of our subsequent focus on stable alliances and in the interest of brevity, the analysis to follow considers only such solutions. Accordingly, the individual's choice in the second-stage satisfies the following equality:

$$\frac{\sum_{i\notin\mathscr{A}_k} m_i}{\left[\sum_{j=1}^{A}\sum_{i\in\mathscr{A}_j} m_i\right]^2}\left[\frac{X}{n_k^2}\right] = 1. \tag{10}$$

Maintaining focus on the case of within-alliance symmetry (i.e., when m_i equals a constant m_j for all $i\in\mathscr{A}_j$ $j=1,2,\ldots,A$), the condition shown in Eq. (10) implies

$$\frac{M - n_k m_k}{M^2}\left[\frac{X}{n_k^2}\right] = 1, \tag{11}$$

where $M\equiv\sum_{j=1}^{A} n_j m_j$.[20] With this condition, one can find the equilibrium effort put forth by each individual belonging to alliance k of size n_k, given the alliance structure, S:

$$m(n_k, S) = [F - (A-1)n_k^2]\frac{(A-1)X}{n_k F^2}, \tag{12}$$

where $F\equiv\sum_{j=1}^{A} n_j^2$ for all k.[21]

In the case where all alliances are of equal size $n\geq 1$, $S\equiv\hat{S}=\{n,\ldots,n\}$,[22] the solution shown in Eq. (12) simplifies to $\hat{m}\equiv m(n,\hat{S})=(N-n)X/N^2n^2$. Under individual conflict where $S=\hat{S}=\bar{S}$, this solution simplifies even further to $\bar{m}\equiv m(1,\bar{S})$ shown in Eq. (5a). By contrast, when the grand alliance forms $n=N$, $m(n,\hat{S})=0$. As can easily be confirmed, under alternative symmetric structures given $N(=An)$, $1\leq n\leq N$, $m(n,\hat{S})$ is decreasing in n or equivalently increasing in A.

For any given alliance structure with $m_k>0$ for all k, the solution for $m(n_k,S)$ reveals more generally that the equilibrium effort by the individual members of alliance k in the inter-

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19 See Lemma 2 in Appendix A.1.

20 Note, however, since the probability of winning X depends on $\sum_{i\in\mathscr{A}_k} m_i$, not just m_i, only total effort by the group is uniquely determined; individual effort, m_i, is not. Although the focus here on the symmetric outcome may make the emergence of alliances more likely, this focus seems most natural given the assumption that individual members of the alliance are identical.

21 Specifically, rewrite Eq. (11) as $X(M-n_k m_k)=M^2 n_k^2$, and sum over all alliances, $k=1,2,\ldots,A$ to obtain $AXM - XM = M^2\sum_{j=1}^{A} n_j^2$. Simplifying and rearranging shows that, in equilibrium, $M=X(A-1)/F$, which with Eq. (11) yields Eq. (12). From this solution, it follows that $m_k>0$ for all k provided that $F>(A-1)n_k^2$ holds for $n_k=n_1$.

22 Ignoring integer problems in this symmetric case, $A=N/n$ and $F=nN$.

alliance conflict is decreasing in the size of the alliance, n_k, as is the total effort by the alliance, $n_k m(n_k,S)$. Not surprisingly, then, the expected probability of winning the conflict in stage 2, given by $\mu_k = [F-(A-1)n_k^2]/F$ for $A>1$, is also decreasing in the alliance size, n_k.

Using this expression for μ_k, Eqs. (9) and (12), the payoff expected by each individual member of alliance k at the end of stage one, $u^e(n_k,S)$, can be written as

$$u^e(n_k S) = [F-(A-1)n_k^2][F-(A-1)n_k]\frac{X}{n_k^2 F^2} \tag{13}$$

for $k=1,2,\ldots,A$, whereas previously defined, $F \equiv \sum_{j=1}^{A} n_j^2$ and $S=\{n_1,n_2\ldots,n_A\}$.

Not surprisingly then, given any alliance structure where $m_k>0$ for $k=1,2,\ldots,A$, individuals belonging to larger alliances expect a smaller payoff than the payoff expected by those belonging to smaller alliances:

$$u^e(n_1,S) \le u^e(n_2,S) \le \ldots \le u^e(n_A,S) \tag{14}$$

where, by assumption $n_1 \ge n_2 \ldots \ge n_A$. Of course, this ranking says nothing about an individual's incentive to move from one alliance to another, as it does not account for the effect of the hypothetical move on the efforts levels m by anyone in the stage-two conflict or others' incentive to move in response. Such incentives are considered more carefully in the analysis of the stable formation of alliances below.

4.3. Expected gains from symmetric alliance formation

Before proceeding to that analysis, this subsection illustrates the gains that individuals could expect under a *symmetric*, *multi-member alliance structure*—i.e., where $n_k=n>1$ for all k: $\hat{S} \equiv \{n,\ldots,n\}$. Using the expression for an individual's expected payoff given in Eq. (13), the expected gains under such an alliance structure relative to the outcome of individual conflict can be written as

$$G^e(n) \equiv u^e(n,\hat{S}) - u^e(1,\bar{S}) = \frac{[(N-n)(n-1)]X}{N^2 n^2} \tag{15}$$

for $n>1$, whereas previously defined $\bar{S} \equiv \{1,\ldots,1\}$.[23] Some straightforward calculations based on this expression reveal the following:

Proposition 1. *Under a symmetric alliance structure, with $n_k=n>1$ $\forall k=1,2,\ldots,A$, the gains expected by each individual, $G^e(n)$, are*

(a) *strictly positive for $n<N$,*
(b) *decreasing in n, and*
(c) *equal to 0 for $n=N$.*

The potential for greater expected payoffs suggests that the cost-saving advantage to appropriative/defense activities by an alliance, which has been highlighted in the literature,

[23] This function is also defined for $n=1$: $G^e(1)=0$. From Eq. (13) under the assumption that $n_k=n$ for all k, one can find $u^e(n,\hat{S})=[N(n-1)+n]X/N^2n^2$. Similarly, $u^e(1,\bar{S})$ can be derived from Eq. (13) assuming $n_k=1$ for all k, which is equivalent to the expected payoff that was derived earlier in Section 3, $\bar{u}_1^e=X/N^2$ (see Eq. (5b)).

might not be essential to the formation of multi-member alliances. In the context of this simple model, the expected gains come in the form of a reduction in the severity of conflict over the contestable resource X for $1<n<N$. No member of an alliance with $n>1$ fully internalizes the benefits of her efforts in that conflict and so naturally devotes less effort to it. In the symmetric outcome, everyone else is doing just the same, so that the net effect on the winning probabilities in the conflict over X relative to the case of individual conflict is zero. Thus, as Proposition 1 indicates, there are potential gains under symmetric alliance formation, with $n<N$.

The proposition also suggests, however, that the expected gains are limited. That is, though positive, the expected gains fall as n rises above 1. As n increases and the second-stage conflict between alliances weakens, the third-stage conflict over the distribution of the product within the alliance intensifies; from an ex ante perspective, the increased costs associated with the intensifying intra-alliance conflict exceed the decreased costs associated with the weakening inter-alliance conflict. As n approaches N, the expected gains from alliance formation go to zero. Of course, the actual outcome under alliance formation with $n=N$ will differ from that under individual conflict by virtue of the difference in the nature of the conflict in the two outcomes. However, by assumption, the alliance has no means by which its members can resolve conflict without resorting to arms (s); therefore, shifting the entire conflict from one level over X to another over Y has no consequences in terms of expected payoffs.[24]

Still, for $n<N$, $G^e(n)>0$ holds, so that the formation of symmetric alliances on net enhances expected welfare. As the next section shows, the expected gain arising from the free-rider problem alone is often sufficient to predict the emergence of alliances in equilibrium.

5. Endogenous alliance formation

Having characterized the allocation of resources in the second and third stages of the game given the alliance structure, consider now the first stage of the game-namely, the formation of alliances in equilibrium. In particular, defining an equilibrium of the first stage as an outcome where no individual can possibly increase her expected payoff, the analysis endogenizes the alliance structure, S. When the number of individuals involved in the second stage conflict, N, is very small, it is possible that only individual conflict emerges in equilibrium. If, however, N is sufficiently large, multiple configurations of stable alliances could emerge in equilibrium.

5.1. Stability and equilibrium

To be sure, in the absence of any specific benefits from belonging to an alliance (i.e., in terms of the conflict or production technologies), there is a strong incentive, given S, for each

[24] If the members of an alliance could credibly agree to share the product equally without arming ($s=0$), the expected payoff under symmetric alliance formation, given in this case by $u^e(n,S)=[N(n-1)+n]X/N^2n$, would be increasing in n, so that the expected gains under alliance formation, $G^e(n)=(n-1)X/Nn$, would also be increasing in n and be strictly positive when evaluated at $n=N$.

individual to break away from her own alliance to form a stand-alone alliance. The logic here is quite simple. As discussed earlier, each member's incentive to contribute to her own alliance's collective effort in the second-stage conflict is decreasing in the size of her alliance. Hence, once having broken away from her alliance to form a stand-alone alliance, given the membership of all other alliances and that of her former alliance, any individual would have an increased incentive to put forth some effort in the conflict over X. At the same time, this deviation would likely decrease the effort made by members of alliances not directly affected by the deviation. By forming a stand-alone alliance, the individual could, then, put herself in a very advantageous position to win the conflict over X and all for herself.

However, suppose that such a deviation must itself be stable or robust to further deviations, which must be stable, and so on. Here the analysis follows the noncooperative theory of endogenous coalition structures which in general imposes certain internal consistency requirements on possible deviations.[25] More specifically, for the purposes of this analysis, we formulate the following definition of an equilibrium:

Definition 1. An alliance structure, $S=\{n_1,n_2,...,n_A\}$, is a Nash equilibrium structure if (i) the payoff expected by each individual under that structure is at least as large as that under individual conflict and strictly larger for at least one individual, and (ii) any deviation from that structure by an individual eventually makes that individual worse off.

Given this definition, the evaluation of the potential gains from a given deviation must factor in the possibility of all subsequent deviations by others and the resulting impact on expected payoffs. In envisioning individuals as looking at the ultimate outcome of a deviation, the equilibrium refinement employed here is most closely related to Chwe's (1994) notion of *farsighted stability*. In the context of this model, although any individual would benefit, for example, by leaving her alliance to form a stand-alone alliance given the membership of the other alliances and her former alliance, such deviations could ultimately trigger a reversion to individual conflict, leaving everyone, including the original deviator, worse off. Accordingly, such deviations themselves would be deemed unprofitable and, thus, would not pose a threat to the stability of the alliance structure under consideration.[26] In effect, invoking the notion of farsighted stability expands the opportunities for "cooperation" among individuals who would behave otherwise in a noncooperative way.

For an open membership game where no consent is required to join an already existing alliance, one must also verify that no individual has an incentive to leave her alliance to join another. From the discussion above, it should be clear that no individual would have an incentive to leave her alliance to join an equal sized or larger alliance (see Section 4.2). However, there may be an incentive to join a smaller alliance. In fact, when the size of the largest alliance exceeds the smallest by 2 or more, each member of the largest alliance, $k=1$, has an incentive, holding the rest of the alliance structure (including her own former alliance $k=1$) fixed, to join one of the smaller alliances. So that no such incentive exists,

[25] See, for example, Bloch (1996), Chwe (1994), Ray and Vohra (1997,1999), and Yi (1997).

[26] Of course, without having specified the dynamics that would take us from a potential deviation to the outcome involving individual conflict, invoking the notion of farsighted stability here might seem ad hoc at best. However, the analysis in connection with Lemmas 1 and 2 in Appendix A.1 is suggestive. Moreover, this stability notion has much theoretical appeal in its emphasis on internal consistency and on the importance of the eventual outcome over the immediate outcome. On these points, see Ray and Vohra (1997, 1999).

an equilibrium alliance structure must be such that the largest alliance have, at most, one more member than any other alliance: $n_1 \leq n_j + 1$ for any $j = 2, \ldots, A$.[27]

5.2. Equilibrium alliance structures

Based on the above discussion, the following characterizes multi-member alliance formation in equilibrium.

Proposition 2. *Fix the number of individuals, N, involved in the conflict over the contestable resource X.*

(a) Suppose N can be decomposed into the product of two integers, $A^*>1$ and $n^*>1$. Then the symmetric multi-member alliance structure with A^* alliances each having n^* members, $\hat{S}^* = \{n^*, \ldots, n^*\}$, is farsighted stable and a Nash equilibrium structure.

(b) Given N, choose any $A^* \in (1,N)$ and define $a \equiv N - A^*n^*$ where $a \in [1,A^*)$. The asymmetric multi-member alliance structure, with a alliances having n^*+1 members and $A^* - a$ alliances having n^* members, $S^* = \{n^*+1, \ldots, n^*+1, n^* \ldots, n^*\}$, is farsighted stable and a Nash equilibrium structure provided n^* satisfies the inequality

$$\frac{[F-(A^*-1)(n^*+1)^2][F-(A^*-1)(n^*+1)]}{(n^*+1)^2F^2} > \frac{1}{(A^*n^*+a)^2}$$

and, in the case that $a = 1$, an additional inequality

$$(A^*n^{*2} - A^*n^* + 1)/(A^*n^{*2}+1)^2 n^{*2} < 1/(A^*n^*+1)^2,$$

where $F = a(n^*+1)^2 + (A^* - a)n^{*2}$.

Proof. See Appendix A.2. □

Part (a) of the proposition establishes that, with the exception of the grand alliance, all symmetric, multi-member alliance structures are stable.[28] Hence, the expected gains from the free-rider problem identified above are sufficient to support the formation of multi-member alliances. Part (b) shows, however, that stability is not unique to symmetric alliance structures. Under asymmetric alliance structures, although the expected gains are unevenly distributed, everyone must be at least as well off as they would be under individual conflict. This requirement, along with the requirement that $n_1 \leq n_j + 1$ for any $j = 2, \ldots, A$, is embedded in the first inequality of the proposition. The inequality ensures further that, for $a = 2, \ldots A^* - 1$ given N, a deviation by one individual originally belonging to a size $n + 1$ group to form a stand-alone group would give at least one other individual the incentive to do the same, thereby inducing a reversion to individual conflict and making the original deviation unprofitable to all. Thus, the first inequality alone is a sufficient condition for the farsighted stability of alliances with $a \in (1,A)$. Matters may

[27] See Lemma 3 in Appendix A.1.

[28] The failure of the grand alliance to emerge as an equilibrium structure is not uncommon in settings where there are positive externalities, especially when N is large (Yi, 1997; Yi and Shin, 2000). The logic is essentially the same here, only more severe because there is no possibility of conflict management within the alliance. Allowing for more peaceful exchange or interaction within the winning alliance, however, would imply $G^e(N)>0$, thereby making the grand alliance a possible outcome, though not necessarily an efficient one (see Garfinkel, 2003).

Table 1
Equilibrium alliance structures

Number of alliances		a	A^*-a	min n^*
$A^*=2$	for $N=2n\geq 4$	0	2	2
	for $N=2n+1\geq 9$	1	1	4
$A^*=3$	for $N=3n\geq 6$	0	3	2
	for $N=3n+1\geq 16$	1	2	5
	for $N=3n+2\geq 11$	2	1	3
$A^*=4$	for $N=4n\geq 8$	0	4	2
	for $N=4n+1\geq 29$	1	3	7
	for $N=4n+2\geq 22$	2	2	5
	for $N=4n+3\geq 11$	3	1	2
$A^*=5$	for $N=5n\geq 10$	0	5	2
	for $N=5n+1\geq 46$	1	4	9
	for $N=5n+2\geq 37$	2	3	7
	for $N=5n+3\geq 23$	3	2	4
	for $N=5n+4\geq 14$	4	1	2
$A^*=6$	for $N=6n\geq 12$	0	6	2
	for $N=6n+1\geq 67$	1	5	11
	for $N=6n+2\geq 56$	2	4	9
	for $N=6n+3\geq 39$	3	3	6
	for $N=6n+4\geq 28$	4	2	4
	for $N=6n+5\geq 17$	5	1	2
$A^*=7$	for $N=7n\geq 14$	0	7	2
	for $N=7n+1\geq 92$	1	6	13
	for $N=7n+2\geq 79$	2	5	11
	for $N=7n+3\geq 66$	3	4	9
	for $N=7n+4\geq 46$	4	3	6
	for $N=7n+5\geq 33$	5	2	4
	for $N=7n+6\geq 20$	6	1	2
$A^*=8$	for $N=8n\geq 16$	0	8	2
	for $N=8n+1\geq 121$	1	7	15
	for $N=8n+2\geq 106$	2	6	13
	for $N=8n+3\geq 91$	3	5	11
	for $N=8n+4\geq 68$	4	4	9
	for $N=8n+5\geq 53$	5	3	6
	for $N=8n+6\geq 38$	6	2	4
	for $N=8n+7\geq 15$	7	1	1

a denotes the number of alliances with n^*+1 members; A^*-a denotes the remaining number of alliances with n^* members. N denotes the total number of individual competing for X.

differ, however, for alliances with $a=1$. Nevertheless, the second inequality serves to rule out the profitability of individual deviations when $a=1$.[29]

Given A^* and for all a, the first equality imposes a lower bound on n. This lower bound on n limits the degree of asymmetry between the size-n groups and the size-$n+1$ groups so as not to give too much of an advantage to the members of the smaller (size-n) groups in the contest for control of X. For example, when $n=3$, the advantage enjoyed by the smaller groups over the larger groups ($n+1=4$) is relatively milder than when $n=2$ (and $n+1=3$).

[29] See Appendix A.2 for more details.

Table 2
Equilibrium alliance sizes given the number of alliances the number of individuals

N	A^*: 2	3	4	5	6	7	8
4	(2,2)						
5							
6	(3,3)	(2,2,2)					
7							
8	(4,4)		(2,2,2,2)				
9	(5,4)	(3,3,3)					
10	(5,5)			(2,2,2,2,2)			
11	(6,5)	(4,4,3)	(3,3,3,2)				
12	(6,6)	(4,4,4)	(3,3,3,3)		(2,2,2,2,2,2)		
13	(7,6)						
14	(7,7)	(5,5,4)		(3,3,3,3,2)		(2,2,2,2,2,2,2)	
15	(8,7)	(5,5,5)	(4,4,4,3)	(3,3,3,3,3)			(2,2,2,2,2,2,2,1)
16	(8,8)	(6,5,5)	(4,4,4,4)				(2,2,2,2,2,2,2,2)
17	(9,8)	(6,6,5)			(3,3,3,3,3,2)		
18	(9,9)	(6,6,6)			(3,3,3,3,3,3)		
19	(10,9)	(7,6,6)	(5,5,5,4)	(4,4,4,4,3)			
20	(10,10)	(7,7,6)	(5,5,5,5)	(4,4,4,4,4)		(3,3,3,3,3,3,2)	
21	(11,10)	(7,7,7)				(3,3,3,3,3,3,3)	
22	(11,11)	(8,7,7)	(6,6,5,5)				
23	(12,11)	(8,8,7)	(6,6,6,5)	(5,5,5,4,4)	(4,4,4,4,4,3)		(3,3,3,3,3,3,3,2)
24	(12,12)	(8,8,8)	(6,6,6,6)	(5,5,5,5,4)	(4,4,4,4,4,4)		(3,3,3,3,3,3,3,3)
25	(13,12)	(9,8,8)		(5,5,5,5,5)			
26	(13,13)	(9,9,8)	(7,7,6,6)				
27	(14,13)	(9,9,9)	(7,7,7,6)			(4,4,4,4,4,4,3)	
28	(14,14)	(10,9,9)	(7,7,7,7)	(6,6,6,5,5)		(4,4,4,4,4,4,4)	
29	(15,14)	(10,10,9)	(8,7,7,7)	(6,6,6,6,5)	(5,5,5,5,5,4)		

Furthermore, increasing the number of larger sized groups ($n+1$) relative to the number of the smaller sized groups (n) (or equivalently when a is larger given A) means that members of the size-$n+1$ groups are put at a relatively smaller disadvantage. Thus, the lower bound on n can be less severe for larger a.[30]

To illustrate these tendencies, Tables 1 and 2 show the stable multi-member alliance structure based on Proposition 2, by the equilibrium number of alliances, A^*.[31] These tables show that, given N and A^*, there need not exist any integer $n>1$ that satisfies the inequality in the proposition. In fact, for $N<4$, $N=5$ and 7, no stable multi-member alliance structure exists. However, as N increases, the conditions for stability, ruling out individual deviations only, weaken considerably. As shown in Table 1, there exists at least one stable multi-member alliance structure for any $N \geq 8$, having $A^*=2$ alliances: for any even number

[30] The second inequality shown in Proposition 2 similarly imposes a lower bound on n, precisely when the constraint imposed by the first inequality is most binding (i.e., when $a=1$). Thus, it might not seem too surprising that, in this setting, the second inequality is implied by the first. However, that is not a general result. The second inequality (given $a=1$) does becomes increasingly relevant over and above the first one once one allows for more peaceful exchange within alliances (see Garfinkel, 2003).

[31] The minimum values of N, or equivalently the minimum values of n given A and $a \in [1,A)$, for which a stable multi-member alliance structure exists were calculated using Mathematica.

Table 3
Expected payoffs under alternative equilibrium structures

N	A:	2	3	4	5	6	7	8	N
9	n	3.44	2.88						1.23
	$n+1$	1.37	2.88						1.23
	N	20.61	25.93						11.11
10	n	1.80			3.00				1.00
	$n+1$	1.80			3.00				1.00
	N	18.00			30.00				10.00
11	n	2.17	5.32	12.36					0.83
	$n+1$	1.03	1.10	1.02					0.83
	N	16.99	24.80	33.87					9.09
12	n	1.27	1.74	2.08		2.43			0.69
	$n+1$	1.27	1.74	2.08		2.43			0.69
	N	15.28	20.83	25.00		29.17			8.33
13	n	1.49							0.59
	$n+1$	0.79							0.59
	N	14.48							7.69
14	n	0.95	2.83		12.00		2.04		0.51
	$n+1$	0.95	0.82		0.78		2.04		0.51
	N	13.27	19.55		33.33		28.57		7.14
15	n	1.08	1.16	4.92	1.63			57.55	0.44
	$n+1$	0.63	1.16	0.78	1.63			0.45	0.44
	N	12.63	17.33	24.12	24.44			63.80	6.67
16	n	0.73	1.48	1.27				1.76	0.39
	$n+1$	0.73	0.39	1.27				1.76	0.39
	N	11.72	17.13	20.31				28.13	6.25
17	n	0.82	1.74			11.78			0.35
	$n+1$	0.51	0.63			0.63			0.35
	N	11.20	16.22			32.99			5.88
18	n	0.58	0.82			1.34			0.31
	$n+1$	0.58	0.82			1.34			0.31
	N	10.49	14.81			24.07			5.56
19	n	0.65	1.01	2.56	4.71				0.28
	$n+1$	0.42	0.34	0.59	0.60				0.28
	N	10.06	14.56	19.07	23.74				5.26
20	n	0.47	1.17	0.85	1.00		11.62		0.25
	$n+1$	0.47	0.49	0.85	1.00		0.53		0.25
	N	9.50	13.89	17.00	20.00		32.76		5.00
21	n	0.52	0.62				1.13		0.23
	$n+1$	0.36	0.62				1.13		0.23
	N	9.14	12.93				23.81		4.76
22	n	0.39	0.74	1.35					0.21
	$n+1$	0.39	0.30	0.27					0.21
	N	8.68	12.68	16.78					4.55
23	n	0.43	0.84	1.55	2.14	4.57		11.51	0.19
	$n+1$	0.30	0.39	0.45	0.21	0.49		0.46	0.19
	N	8.37	12.17	15.86	20.28	23.50		32.59	4.35
24	n	0.33	0.48	0.61	2.42	0.82		0.98	0.17
	$n+1$	0.33	0.48	0.61	0.46	0.82		0.98	0.17
	N	7.99	11.46	14.58	18.79	19.79		23.61	4.17

Table 3 (*continued*)

N	A:	2	3	4	5	6	7	8	N
25	n	0.36	0.56		0.67				0.16
	$n+1$	0.26	0.25		0.67				0.16
	N	7.72	11.23		16.80				4.00
26	n	0.28	0.63	0.91					0.15
	$n+1$	0.28	0.32	0.24					0.15
	N	7.40	10.83	14.26					3.85
27	n	0.31	0.38	1.03			4.47		0.14
	$n+1$	0.23	0.38	0.36			0.41		0.14
	N	7.16	10.29	13.62			23.33		3.70
28	n	0.25	0.44	0.46	1.28		0.70		0.13
	$n+1$	0.25	0.22	0.46	0.21		0.70		0.13
	N	6.89	10.08	12.76	16.58		19.64		3.57
29	n	0.26	0.49	0.56	1.44	2.32			0.12
	$n+1$	0.20	0.27	0.12	0.35	0.37			0.12
	N	6.68	9.76	12.70	15.66	18.62			3.45

The first seven columns ($A=2-8$) report the expected payoffs under the equilibrium multi-member alliance structures reported in Table 2. The last column ($A=N$) reports the analogous expected payoffs under individual conflict. The entries for n and $n+1$ report the payoffs expected by each member of groups having, respectively n and $n+1$ members. For symmetric groups, the same payoff is indicated for both groups. The entry for N reports the expected payoffs summed over all individuals. [The individual payoffs might not sum to the aggregate payoffs due to rounding.] These calculations assume that $X=100$.

$N\geq 8$, both alliances have $n^*=N/2$ members; and, for any odd number $N>8$, one alliance has $n^*=(N-1)/2$ members and the other has just one additional member, $n^*+1=(N+1)/2$. Yet, as clearly shown in Table 2, other alliance structures are also possible. In general, for any given $A^*>2$, alliance structures with fewer size-$n+1$ alliances (or smaller a) are more likely to be farsighted stable when N is larger.[32]

With the multiplicity of possible structures, one might naturally wonder how they would be ranked among the participants.[33] For $N<9$, where there are multiple equilibrium structures, they are all symmetric. Hence, from Proposition 1, it is clear that everyone would prefer the structure having the greatest number of alliances, $A^*=N/n^*$ or equivalently the smallest number of alliance members, $n^*>1$. In the case where both asymmetric and symmetric alliances are possible, the ranking is not so obvious. Table 3 reports the expected payoffs per individual in each alliance, under the alternative equilibrium alliance structures listed in Table 2, for $N\geq 9$. It also reports aggregate expected payoffs under each of those structures and under individual conflict. First, the table confirms that, under asymmetric alliance structures, the benefits of alliance formation relative to individual conflict are

[32] Though not shown here, one can verify that, for all $N\geq 15$, there exists yet another alliance structure: for even $N>15$, there is one stable structure with $A^*=N/2$ alliances, each having two members; and, for odd $N\geq 15$, there exists one stable structure with $A^*=(N+1)/2$ alliances, the first $a=A^*-1$ each having two members and the last one having just one member. This is the least concentrated alliance structure that can possibly emerge in equilibrium. For any structure with two or more alliances having just one member cannot be stable (see Appendix A.1).

[33] One could apply the equilibrium refinement introduced by Bernheim et al. (1987), accounting for deviations of groups of individuals as well as for individual deviations, and thereby sharpen the predictions of the analysis in terms of the number of alliances and the size of alliances given N. However, there need not be any equilibrium at all (for an example, see Yi and Shin, 2000) In any case, such an analysis is beyond the scope of the present paper.

distributed asymmetrically.[34] When more than one stable multi-member alliance structure exists for a given N, the payoffs expected by members of the size-n alliances increase unambiguously as we move to another asymmetric alliance structure with a larger number of alliances, A^*. The same cannot be said for members of size-$n+1$ alliances. While in some cases, their expected payoffs increase as well, sometimes they fall but never below what would be expected under individual conflict. Furthermore, as we move from an asymmetric alliance structure to a symmetric one with a greater number of alliances (A), the payoffs expected by each member of a size-n alliance fall while those expected by each member of a size-$n+1$ alliance rise. Nonetheless, note that regardless of whether the alliance is symmetric or asymmetric, aggregate expected payoffs are unambiguously rising in the number of alliances (A) for any given N.[35]

These implications would appear to contrast sharply with those of Baik and Lee's (2001) equilibrium analysis of strategic group formation with *individual* rent-seeking. In that analysis, individuals win or lose the contest on their own; if they win, they either share their prize with other members of the group or receive an extra payoff from them. The particular sharing rule is chosen by the groups' members before the conflict. When more than two (multi-member) strategic groups form in which case the groups must be of a similar size as in the present analysis, within-group optimality dictates that the winner takes the entire prize for himself and a bonus from the other members of his alliance. In this case, though better off as a member of a group, everyone is worse off than they would be under individual conflict, and overall welfare is decreasing in the number of strategic groups. Such outcomes, however, are dominated by the one where just one (multi-member) strategic group forms and all other individuals participate in the conflict on their own.[36] Within-group optimality requires, in this case, that the winner shares his prize with the other members of his group, such that the free-rider problem comes into play, as in the present analysis, to reduce the overall level of conflict. Hence, more in line with the predictions of the present analysis, strategic group formation enhances everyone's expected payoffs. That the equilibrium structure in this case looks very different from that in the present analysis can be attributed to Baik and Lee's assumption that members within a group can somehow commit to a sharing rule—that is, to allow for some form of cooperation.[37]

[34] Observe from both Tables 2 and 3, consistent with the previous discussion, that for any given A^* and a, expected payoffs are more evenly distributed for larger N. Furthermore, given N, they tend to be more evenly distributed when the number of size-$n+1$ alliances (a) is larger relative to A^*.

[35] A close examination of the table suggests that the benefits of group-size asymmetry, as identified by Katz and Tokatlidu (1996), can be realized only by increasing the number of groups. That is to say, group-size asymmetry is not the sole source of the benefit. In fact, Katz and Tokatlidu who fix the number of groups to 2 find that the rate of rent dissipation is non-monotonic in group size asymmetry. Hence, if one group is sufficiently larger than the other, then increasing that group's membership any further has a negative effect on aggregate expected payoffs.

[36] Note that, when $N \leq 5$, the grand coalition can emerge in equilibrium, in which case the social waste associated with the conflict is entirely eliminated. For $N > 5$, the equilibrium size of the group is the smallest integer greater than $N/2$.

[37] Also see the recent analysis of Bloch et al. (2002), who find that the grand alliance emerges as the unique equilibrium of a sequential group formation game. The strong tendency of grand alliance to emerge in equilibrium there, despite the presence of positive externalities as in the present analysis, appears to be driven, in large part, by the assumption that members of the alliance share the "prize" equally. That is to say, the analysis abstracts from the difficulties of intra-alliance conflict.

6. Concluding remarks

This paper has investigated the formation of stable alliances under rather restrictive conditions. One central finding is that increasing returns in neither the conflict technology nor the production technology is essential for the stability of alliances. Furthermore, there need not be any possibility for peaceful exchange or interactions within the alliance. Instead, the effect of alliances to reduce the degree of conflict over the contestable resource alone could be sufficient to support their stability in equilibrium.

This is not to say that these other factors related to the technology of conflict and production are irrelevant. Indeed, the analysis of this paper has deliberately abstracted from many of the features of alliances that might help to explain their emergence in more "civilized" settings. The central objective here was to study their formation in the most primitive environment possible. Natural extensions of the present analysis, left for future research, then, would be to consider these features.

Consider, for example, the process by which members within a group resolve a given sort of conflict. It would seem reasonable to suppose, contrary to the assumption of this paper, that the survival of groups over time requires the creation and maintenance of "norms" and institutions that would allow the alliance members to effect a more "peaceful" distribution of output at a lower cost. More specifically, within the context of this model, the distribution of output should not depend entirely on force or security measures taken by the groups' members to guard against one another. In a modest extension of the present analysis, Garfinkel (2003) incorporates the possibility of conflict management, allowing for varying degrees of "cooperation" within alliances.[38] Given the number of individuals in competition for the contestable resource, when mechanisms of conflict management are relatively more important in determining the distribution of the group's joint product, a greater variety of group structures are possible in equilibrium. When such mechanisms are sufficiently effective in conflict resolution, larger groups are more likely to emerge in equilibrium relative to what has been suggested in the present analysis. However, provided that some within-group conflict remains despite whatever mechanisms of conflict management are in place, larger groups need not be better. That is to say, the grand alliance generally is not the efficient outcome.

A related extension would involve relaxing the assumption that individuals are ex ante identical. While heterogeneity within the population (in terms of technologies and preferences) would seem more reasonable in an analysis of group formation, most analyses assume ex ante symmetry. Indeed, Skaperdas (1998) and Noh (2002) show just how complex the problem quickly becomes in the case of only three individuals having different endowments. In the present analysis, without ex ante symmetry, it is no longer possible to characterize an alliance structure by the number and size of alliances alone. Nevertheless, such an extension seems worth pursuing, as the heterogeneity of individuals raises some important and interesting issues about the composition of alliances and about resolving conflicts therein, provided that a stable structure exists at all.

[38] Genicot and Skaperdas (2002) go somewhat deeper, modelling conflict management as an investment decision in a dynamic setting.

Another interesting extension focuses on the technology of conflict. In abstracting from any sort of technological advantage that may exist for those who join forces in an alliance [e.g., super-additivity, as in the work of Skaperdas (1998)], the analysis has emphasized the positive externalities of their formation. Specifically, in the context of the present framework, when a new alliance forms or merges with another, those belonging to the new (larger) alliance have a smaller incentive to compete for the contestable resource, implying that everyone else enjoys a higher likelihood of success in the contest and thus a higher expected payoff. As a result, conflict over the contestable resource falls.[39] A conflict technology exhibiting increasing returns and negative externalities could yield very different implications.[40] In particular, the effect of increasing returns on the incentive of individuals to contribute to the collective effort could swamp the effects of the free-rider problem as the size of the alliance rises, in which case group formation might aggravate the conflict.

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Appendix A

A.1. Preliminary results for stability and equilibrium

Lemma 1. *Given an alliance structure with two or more stand-alone alliances $S=\{n_1,\ldots,n_{A-2},1,1\}$, all members $i\in\mathcal{A}_k$ where $n_k\geq 2$ optimally choose not to participate in the second-stage conflict: $m(n_k,S)=0$.*

Proof. By hypothesis, $n_{A-1}=n_A=1$, implying that $F=\sum_{k=1}^{A-2}n_k^2+2$. Even if all of the other (multi-member) alliances were of the same size, such that F would be equal to the largest possible value of $F=(A-2)n_1^2+2$, the inequality, $F-(A-1)n_1^2<0$, would hold, implying by Eq. (12) $m(n_1,S)=0$. Now, consider alliance $k=2$'s decision to participate. Since alliance $k=1$ is not active in the conflict over X, $F=\sum_{k=2}^{A-2}n_k^2+2$. As before, even when F is equal to the largest possible value, $F'=(A-3)n_2^2+2$, the inequality, $F'-(A-2)n_2^2<0$, would hold, implying $m(n_2,S)=0$. With repeated applications of this logic, given S, one can show sequentially that the remaining alliances $k\geq 3$ for which $n_k\geq 2$ have no incentive to participate in the conflict over X. □

[39] Of course, the new hypothetical alliance structure need not be stable.
[40] See Yi (1997) for a useful discussion of endogenous coalition formation games with positive and negative externalities.

Lemma 2. *Stability of any given alliance structure, $S=\{n_1,n_2,\ldots,n_A\}$, requires $m_k>0$, for $k=1,2,\ldots,A$.*

Proof. Suppose there exists an alliance structure, S', in which the members of one alliance $i\in\mathscr{A}_k$ have no incentive to participate in the conflict, $m(n_k,S')=0$. From Eq. (12), this alliance must be the largest one: $k=1$. By hypothesis, each $i\in\mathscr{A}_1$ would obtain a payoff of just zero. Yet, given the membership of all other alliances, any individual $i\in\mathscr{A}_1$ could obtain a higher expected payoff by competing for the contestable resource on her own, giving a new partition $S''=S'\setminus\{n_1\}\cup\{n_1-1,1\}$.[41] Hence, the original structure could not have been stable. Assuming that this deviation does not affect the participation decision of the remaining members of alliance $k=1$ ($m(n_1-1,S'')=0$), the new structure is not stable either.[42] Here, there are two general sets of circumstances to consider:

Case 1. $n_k=1$, for $k\leq A$. If before the initial deviation there had been at least one other stand-alone alliance, by Lemma 1, the initial deviation would push the all individuals belonging to a multi-member alliance (k for $n_k\geq 2$) out of the conflict, $m(n_k,S'')=0$.

Case 2. $n_A>1$. Each of the remaining members of alliance $k=1$ could obtain a higher payoff by competing for the contestable resource on her own as before. But then from Lemma 1, a move by any one of them would result in another alliance structure S''' such that anyone remaining in a multi-member alliance (k for $n_k\geq 2$) would pull out of the conflict, $m(n_k,S''')=0$.

In either case, any individual $i\in\mathscr{A}_k$ with $n_k>1$ and thus $u^e(n_k,S)=0$ would have an incentive to deviate from the existing alliance structure, S'' in case 1 and S''' in case 2. In the very least, each could leave her alliance to form a stand-alone alliance and, regardless of others' choices, expect a positive payoff equal to $u^e(1,\tilde{S})=X/\tilde{N}^2$ where $\tilde{S}$ consists of $\tilde{N}\leq N$ singleton alliances and $N-\tilde{N}\geq 0$ individuals belonging to one or more multi-member alliances. Given a zero payoff for non-participation, the incentive for any individual to move from her current multi-member alliance to form another new single-member alliance would remain strictly positive.[43] □

Lemma 3. *When the size of the largest alliance exceeds the smallest by 2 or more, any member of the largest has an incentive to join one of the smaller alliances, holding the rest of the alliance structure (including the remainder of the largest alliance) fixed: $u^e(n_1,S)<u^e(n_j+1,S')$, where $n_1>n_j+1$ and $S'=S\setminus\{n_1,n_j\}\cup\{n_1-1,n_j+1\}$.*

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[41] To be more precise, under S'' assuming that all other alliances, $k\geq 2$, remain active in the stage-two conflict, $F''=F'+1$ where $F'=\sum_{k=2}^{A}n_k^2$. Then, from Eq. (13), $u^e(1,S'')=(F''-(A-1))^2X/F''^2>0$.

[42] Members of even smaller alliances $k\geq 2$ might pull out of the conflict for an expected payoff of zero too. That would not change the basic logic of the argument to follow. If instead $m(n_1-1,S'')>0$, the proof would be complete.

[43] Given the focus on individual deviations in the analysis of equilibrium, it seems reasonable to conjecture a tendency towards individual conflict.

Proof. Using Eq. (13), it is necessary to verify that the following inequality holds:

$$\frac{X[F-(A-1)n_1^2][F-(A-1)n_1]}{n_1^2F^2} < \frac{X[F'-(A-1)(n_j+1)^2][F'-(A-1)(n_j+1)]}{(n_j+1)^2F'^2} \tag{A.1}$$

for $n_1 > n_j+1$, where $F = n_1^2 + n_j^2 + \mathrm{B}$, $F' = (n_1-1)^2 + (n_j+1)^2 + B$ and $B = \sum_{k \neq 1,j} n_k^2$. Assume that, under both alliance structures, $m_k > 0$ for all k.[44] Some tedious algebra shows that the inequality above will be satisfied if and only if the following condition is satisfied:

$$\begin{aligned}&[(n_j+1)^2F'^2 - n_1^2F^2][(F-(A-1)n_1^2)(F-(A-1)n_1)] \\ &\quad < n_1^2F^2[[F'-(A-1)(n_j+1)^2][F'-(A-1)(n_j+1)] \\ &\quad - [F-(A-1)n_1^2][F-(A-1)n_1]].\end{aligned} \tag{A.2}$$

Note that if $n_1 = n_j+1$, then $F = F'$ and the two sides of the expression are identical and equal to 0. However, the assumption that $n_1 > n_j+1$ implies $F = F' + 2(n_1 - n_j - 1) > F'$, making the left hand side of Eq. (A.2) negative. Thus, a sufficient condition for Eq. (A.1) to hold for $n_1 > n_j+1$ is that the right hand side of Eq. (A.2) be positive. More tedious algebra shows that the right hand side of Eq. (A.2) is positive for $n_1 > n_j+1$ if and only if,

$$\begin{aligned}&n_1^2F^2(n_1-n_j-1)\big[(A-3)[F-(A-1)n_1^2] \\ &\quad + [(A-1)(n_1+n_j)+(A-3)][F'-(A-1)(n_j+1)]\big] > 0.\end{aligned} \tag{A.3}$$

When $A \geq 3$, the inequality clearly holds. When $A = 2$, there are just two alliances and the term B vanishes from F and F'. In this case, more tedious algebra shows that the expression above simplifies as follows:

$$n_1^2(n_1^2+n_j^2)^2(n_1-n_j-1)[(n_1+n_j-1)[(n_1-1)^2+n_j^2]+n_j(n_1-1)], \tag{A.4}$$

which is clearly positive. Therefore, in equilibrium, the difference in the sizes of any two alliances cannot be greater than 1; it must be 0 or 1. □

A.2. Proof of Proposition 2

Part a: symmetric alliances. By Proposition 1, the expected payoffs under $\hat{S}^*$ for $A^* > 1$ and $n^* \geq 2$ are strictly greater than those under individual conflict, $\bar{S}$. Hence, any deviation that triggered a reversion to individual conflict would be considered unprofitable relative

[44] Thus, $F > (A-1)n_k^2$ and $F' > (A-1)n_k^2$ for all k.

to $\hat{S}^*$. Now suppose an individual were to leave her alliance to form a stand-alone alliance. Then there would be A^*+1 alliances: A^*-1 alliances of size n^*, the deviator's former alliance of size n^*-1, and the deviator's new single-member alliance. As one can verify using Eq. (12), given the membership of the deviator's former alliance ($n^*-1 \geq 1$), such a deviation from the symmetric structure would push the members of the other original A^*-1 alliances to the corner $m=0$, implying a reversion to individual conflict by Lemma 2. Finally, since alliances are all of the same size and $n^* \geq 2$ under $\hat{S}^*$, no individual would have an incentive to leave her alliance to join another (Lemma 3). □

Part b: asymmetric alliances. By construction, under S^* each alliance is of size n or $n+1$, such that no individual has an incentive to join another alliance (Lemma 3). The inequality ensures, in addition, that the expected payoff under that structure for any member belonging to a size-$n+1$ alliance is greater than that under individual conflict. Then, by Eq. (14), members of all alliances would consider any deviation which triggered a reversion to individual conflict to be unprofitable relative to S^*. Thus, to verify that no individual would have an incentive to form an alliance on her own, it is only necessary to show that the payoffs expected by another under that hypothetical deviation are less than that under individual conflict. For $a=2,\ldots, A^*-1$, given N, such a deviation would result in a new partition with $A+1$ groups: A^*-a+1 groups with n^* members, $a-1$ groups with $n+1$ members and 1 stand-alone group, implying that $F–A(n+1)^2 = -(A-a)(1+2n)-2n<0$. Thus, from Eq. (12), this individual deviation would push the members of the remaining $a-1$ groups with $n+1$ members to the corner ($m=0$) for a zero payoff. By the reasoning of Lemma 2, such a deviation would trigger a reversion to individual conflict and would therefore be deemed unprofitable. For $a=1$, when an individual from the single group of size $n+1$ forms a group on her own, a new structure, again with $A+1$ groups, emerges: A groups of size n and one stand-alone group. In this case, such a deviation would not push anyone away from an interior optimum. However, as long as the second inequality shown in the proposition holds, everyone but the original deviator would be worse off than if there were no groups at all. Then, by the reasoning applied earlier, the initial deviation would induce a reversion to individual conflict, making everyone, including the original deviator, worse off relative to the initial alliance structure under consideration. □

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