STABILITY AND THE DISTRIBUTION OF POWER

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The relation between the distribution of power and the probability of war has been at the center of a long and important debate in international relations theory. Is an even distribution of power more or less stable than a preponderance of power? The balance-of-power school generally argues that an even distribution of power is more stable, whereas the preponderance-of-power school generally argues the opposite, that a preponderance of power is more stable.1

Empirical efforts to resolve this debate have yielded equivocal results. Siverson and Tennefoss, for example, find an even distribution of power to be more stable, whereas Kim and Moul find a preponderance of power to be more stable. Singer, Bremer, and Stuckey find an even distribution to be more stable than a preponderance of power during the nineteenth century and the opposite during the twentieth century. Maoz and Bueno de Mesquita and Lalman, by contrast, find no significant relation between the probability of war and the distribution of power. Finally, Mansfield finds evidence of a nonlinear, quadratic relation in which the probability of war is smallest when there is an even distribution and a preponderance of power.2

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Claims about the relation between the distribution of power and the likelihood of war have generally lacked any formal foundations. Some recent work has attempted to fill this gap, however. Bueno de Mesquita and Lalman and Fearon analyze models of international bargaining. The former find, first, that the probability of war is smallest when the distribution of power is uneven and, second, that "dissatisfaction with the status quo is unrelated to the likelihood of war."\(^3\) Fearon, by contrast, finds that the probability of war is independent of the distribution of power; that is, a change in the distribution of power has no effect on the likelihood of war.\(^4\)

This essay examines the relation between the probability of war and the distribution of power in the context of a game-theoretic model in which two states are bargaining about revising the international status quo. In the model the states make proposals for changing the status quo. The bargaining continues until the states reach a mutually acceptable settlement or until one of them becomes sufficiently pessimistic about the prospects of reaching an agreement that it uses force to try to impose a new international order. The states’ equilibrium strategies specify the demands the states will make of each other and the circumstances in which they will resort to force. The equilibrium strategies thus make it possible to calculate the probability that bargaining will break down in war as a function of the distribution of power between the two states. The shape of this function can then be compared to the relation claimed to exist by the balance-of-power and preponderance-of-power schools.

The probability of war in the model turns out to be a function of the disparity between the status quo distribution of benefits and the distribution expected to result from the use of force. The outcome expected from using force is, in turn, directly related to the distribution of power.

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\(^3\) Bueno de Mesquita and Lalman (fn. 2, 1992), 204–5, 190.

Thus, the probability of war is a function of the disparity between the status quo and the distribution of power. When this disparity is small, the division of benefits expected from the use of force is approximately the same as the existing status quo distribution. The gains to using force are therefore too small to outweigh the costs of fighting. Neither state is willing to use force to change the status quo, and the probability of war is zero. When the disparity between the status quo division of benefits and the distribution of power is large, then at least one state is willing to use force to overturn the status quo. Moreover, as the disparity grows, the probability of war generally increases.

These results seem quite intuitive: as the disparity between the distribution of power and the distribution of benefits grows, the probability of war generally increases. Indeed, this formal result echoes Gilpin’s explanation of hegemonic wars. Such wars arise, he argues, because of a disparity between the distribution of power and the international status quo distribution of benefits imposed by the hegemon following the previous hegemonic war. Although intuitive, these results contradict the expectations of both the balance-of-power and preponderance-of-power schools. The former expects the probability of war to be smallest when power is evenly distributed, but the probability of war is smallest in the present model when the distribution of power mirrors the status quo distribution of territory. These claims will coincide only in the special case in which the status quo distribution is even. The preponderance-of-power school expects the probability of war to be smallest when the distribution of power is highly skewed and to increase as the distribution of power becomes more even. But the probability of war is largest in the model developed below when the distribution of power is highly skewed.

The formal results derived here also differ from those obtained by Fearon and Bueno de Mesquita and Lalman. Unlike Fearon, the probability of war in the present formulation does vary with changes in the distribution of power. Unlike Bueno de Mesquita and Lalman, who find that “dissatisfaction with the status quo is unrelated to the likelihood of war” and that the probability of war declines as the distribu-

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2 Although the potential use of force is not the source of the coercive pressure, some suggest that international regimes are more likely to break down when the underlying distribution of power differs from the distribution of benefits the regime confers. See Robert Keohane and Joseph Nye, Power and Interdependence (Boston: Little Brown, 1977), 139; and Stephen Krasner, “Regimes and the Limits of Realism,” in Krasner, ed., International Regimes (Ithaca, N.Y.: Cornell University Press, 1983), 357–58.
3 Fearon (fn. 4).
tion of power becomes more skewed, the probability of war is directly related to the level of dissatisfaction with the status quo and generally increases as the distribution of power becomes more skewed.8

The remainder of this essay is organized as follows. The next section motivates and defines the model and discusses some qualifications and limitations. The subsequent section describes the game's equilibrium. The final section uses this equilibrium to specify the probability of war as a function of the distribution of power and compares these results with the expectations of the balance-of-power and preponderance-of-power schools and with other formal work. That section also offers a possible explanation for the equivocal empirical results of efforts to measure the relation between stability and the distribution of power.

THE MODEL

To motivate the model, suppose two states are bargaining about revising the international status quo. More concretely, imagine that two states are negotiating about changing the territorial status quo. Each state proposes a division of the territory that the other state can accept or reject. The bargaining continues with offer followed by counteroffer until the states agree on a proposed division or until one of the states becomes sufficiently pessimistic about the prospects of reaching a mutually acceptable settlement that it resorts to force to try to impose a new territorial settlement.9

How do changes in the distribution of power between these two states affect the bargaining and, more specifically, the probability that the bargaining will break down in war? Are the negotiations more likely to break down if one state has a preponderance of power or if there is an even distribution of power? It might seem at first that the preponderance-of-power school is clearly correct and that the bargaining is less likely to break down if one of the states has a preponderance of power. That is, suppose a state, say $S_1$, has just proposed a particular division to the other state $S_2$. $S_2$ would be tempted to use force to impose a settlement rather than accept $S_1$'s offer only if the expected payoff to using force were at least as large as the payoff to agreeing to the proposal. But the weaker $S_2$, the lower its expected payoff to fighting.

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8 Bueno de Mesquita and Lalman (fn. 2, 1992), 190, 204–5.

9 In a more general model the two states might be bargaining about altering the international status quo where the set of feasible outcomes is an $n$-dimensional policy space. Each point in this space represents a different international order and a different distribution of benefits for the two states. The formal analysis developed below applies equally well to this more general formulation.
deed, if $S_2$ is sufficiently weak, its payoff to using force will be less than its payoff to agreeing to $S_1$'s offer. In this case, $S_2$ will acquiesce to $S_1$'s demand and will not be tempted to use force. Nor will $S_2$ be tempted to use force, since $S_2$ is willing to agree to $S_1$'s demand. This reasoning, which suggests that a preponderance of power will be more stable, is essentially the reasoning Organski advances: “A preponderance of power . . . increases the chances of peace, for the greatly stronger side need not fight at all to get what it wants, while the weaker side would be plainly foolish to attempt battle for what it wants.”

There is, however, a countervailing factor at work. The preceding argument was based on the assumption that $S_1$'s proposal was fixed. But as $S_1$ becomes stronger and $S_2$ becomes weaker, $S_1$ knows that $S_2$ is more likely to accept any given proposal. Thus, $S_1$ will demand more of $S_2$ as $S_2$ becomes weaker. $S_2$, in turn, may be more likely to resist these larger demands, and war may become more likely when there is a preponderance of power. If so, then a preponderance of power would be less stable. By contrast, an even distribution of power would restrain all states' demands and thereby enhance stability. This restraining effect of an even distribution of power underlies the balance-of-power school's analysis of the relation between stability and the distribution of power. As Wolfers argues, “Thus, from the point of view of preserving the peace . . . it may be a valid proposition that a balance of power placing restraint on every nation is more advantageous in the long run than the hegemony even of those deemed peace-loving at the time.”

In sum, a shift in the balance of power against $S_2$ has two competing effects. First, $S_2$ is more likely to accept any specific demand. But, second, more will be demanded of it than would have been had it been more powerful. The first effect tends to make war less likely, while the second tends to make war more likely. The net result of these two opposing effects on the probability of war is unclear: whereas the preponderance-of-power school emphasizes the first effect, the balance-of-power school underscores the second effect. The game-theoretic model of bargaining developed here makes it possible to weigh the interaction of these two competing effects more formally.

Rubinstein's seminal analysis of an alternating-offer, infinite-horizon bargaining game provides a point of departure for the present analysis. In Rubinstein's model two actors are trying to divide a pie. One actor begins the game by proposing a division of the pie to the other.

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10 Organski (fn. 1), 294.
11 Wolfers (fn. 1), 120. See also Claude (fn. 1), 62.
who can either accept or reject the offer. Acceptance ends the game and the pie is divided as agreed. If the second actor rejects the initial proposal, it makes a counteroffer that the first actor can either accept or reject. Acceptance again ends the game. If the first actor rejects the proposal, it makes a new offer. The actors alternate making offers until they agree on a division.13

The present model adds the option of using force to Rubinstein’s basic structure. As illustrated in Figure 1, \( S_1 \) begins the game by proposing a division of the status quo. More formally, \( S_1 \) demands some fraction \( x \) of the disputed territory where \( x \) lies between zero and one. If, for example, \( S_1 \) sets \( x = 1 \), then \( S_1 \) is demanding all of the territory for itself; if \( S_1 \) sets \( x = 1/2 \), it is demanding half of the territory; and if \( S_1 \) sets \( x = 0 \), it is offering \( S_2 \) all of the territory. (The arc between “0” and “1” in Figure 1 indicates that a state can demand any fraction between 0 and 1.) \( S_2 \) can accept this offer by saying yes, which is denoted by \( Y \); reject it by saying no, \( N \), and subsequently make a counteroffer; or force a settlement by attacking \( S_1 \) with \( A \). The game ends if \( S_2 \) accepts or attacks. \( S_1 \) rejects, it then proposes a new territorial division. \( S_1 \) can now accept, reject, or attack. As before, accepting or attacking ends the game. If \( S_1 \) rejects, it again suggests another division. The bargaining continues, possibly forever, with offers alternating back and forth until one of the states accepts a proposal or attacks.

To specify the states’ payoffs, let \((q, 1 - q)\) denote the status quo division where \( q \) and \( 1 - q \) are \( S_1 \)’s and \( S_2 \)’s respective shares. Now suppose that the states agree to divide the territory according to \((x, 1 - x)\) where \( S_1 \) and \( S_2 \) receive \( x \) and \( 1 - x \), respectively. Assume further that this agreement is reached at time \( t \), that is, after \( t + 1 \) offers. (For notational convenience, the first offer is made at time \( t = 0 \).) If agreement is reached at time \( t \), then \( S_1 \) had \( q \) and \( S_2 \) had \( 1 - q \) from the beginning of the game until time \( t - 1 \). The payoff that \( S_1 \) derives from having a flow of \( q \) for the first \( t \) periods will be taken to be \( U_1(q) + \delta U_1(q) + \delta^2 U_1(q) + \cdots + \delta^{t-1} U_1(q) = (1 - \delta^t)U_1(q)/(1 - \delta) \) where \( \delta \) is the states’ common discount factor and \( U_1 \) is \( S_1 \)’s utility function for benefits. \( S_1 \) will also be assumed to prefer larger allocations to smaller allocations but with a nonincreasing marginal value.14 Once agreement is reached at time \( t \), \( S_1 \)’s allocation from time \( t \) forward is \( x \). The present value of having \( x \) from time \( t \) forward is \( \delta U_1(x) + \delta^t U_1(x) + \delta^{t+1} U_1(x) + \cdots = \delta^t U_1(x)/(1 - \delta) \). Thus, \( S_1 \)’s payoff to agreeing to \((x, 1 - x)\) at time \( t \) is \((1 - \delta^t)U_1(q)/(1 - \delta) + \delta^t U_1(x)/(1 - \delta) \). Similarly, \( S_2 \)’s payoff to agree-

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13 Rubinstein (fn. 12) showed that this game has a unique subgame-perfect equilibrium.

14 More formally, \( U_1 \) is assumed to be twice differentiable with \( U_1' > 0 \) and \( U_1'' \leq 0 \). \( S_1 \) is risk neutral if \( U_1'' = 0 \) and risk averse if \( U_1'' < 0 \).
FIGURE 1
AN INFINITE-HORIZON, ALTERNATING-OFFER GAME WITH THE OPTION OF USING FORCE
ing to \((x, 1 - x)\) at time \(t\) is
\[
(1 - \delta^t)U_1(q)/(1 - \delta) + \delta U_2(1 - x)/(1 - \delta)
\]
where \(U_2\) is \(S_2\)'s utility.

To define the states' payoffs if they fail to reach an agreement and one of them uses force, suppose that they fight at time \(t\). As before, \(S_1\) derives \((1 - \delta^t)U_1(q)/(1 - \delta)\) from having \(q\) during the first \(t\) periods. To simplify the specification of \(S_1\)'s payoff following an attack, war will be assumed to end in one of only two ways. Either \(S_1\) will be completely victorious, or \(S_2\) will be. (The limitations of this simplification are discussed below.) If \(S_1\) prevails, it takes all of the territory at a cost of fighting \(c_1\). The payoff to having all of the territory less the cost of fighting from time \(t\) forward is
\[
\delta^t(U_1(1) - c_1)/(1 - \delta) + \delta^{t+1}(U_1(1) - c_1) + \delta^{t+2}(U_1(1) - c_1) + \ldots = \delta^t(U_1(1) - c_1)/(1 - \delta)
\]
If \(S_1\) loses, \(S_2\) captures all of the territory which leaves \(S_1\) with nothing but the cost of fighting. The payoff to this outcome is
\[
\delta^t(U_1(0) - c_1)/(1 - \delta)
\]
Going to war is an uncertain venture. Let \(p\) reflect the balance of power or the distribution of capabilities between \(S_1\) and \(S_2\). That is, \(p\), which is assumed to be common knowledge, is the probability that \(S_1\) will prevail if force is used. Then \(S_1\)'s expected payoff to a bargaining process that ends in war at time \(t\) is the value of living with the status quo until time \(t - 1\) plus the payoff to prevailing at time \(t\) times the probability of prevailing, \(p\), plus the payoff to losing at time \(t\) times the probability of losing, \(1 - p\):
\[
\frac{(1 - \delta^t)U_1(q)}{1 - \delta} + p \frac{\delta^t(U_1(1) - c_1)}{1 - \delta} + (1 - p) \frac{\delta^t(U_1(0) - c_1)}{1 - \delta}
\]
This payoff can be simplified by normalizing the utility function by setting the payoff to having all of the territory equal to one and the payoff to having no territory equal to zero (that is, \(U_1(1) = 1\) and \(U_1(0) = 0\)). Letting \(W_1(t)\) and \(W_2(t)\) denote \(S_1\)'s and \(S_2\)'s normalized payoffs to a bargaining process that ends in war at time \(t\) leaves:
\[
W_1(t) = \left(\frac{1 - \delta^t}{1 - \delta}\right) U_1(q) + \frac{\delta^t}{1 - \delta} (p - c_1)
\]
\[
W_2(t) = \left(\frac{1 - \delta^t}{1 - \delta}\right) U_2(1 - q) + \frac{\delta^t}{1 - \delta} (1 - p - c_2)
\]

\(^{15}\text{There are no offensive or defensive advantages in the present formulation, so it makes no difference whether }S_1\text{ or }S_2\text{ attacks.}\)
The model developed so far has complete information. Each state knows the other state's payoffs. As discussed more fully below, as long as information is complete, bargaining never breaks down, regardless of the distribution of power. In this model incomplete information is a prerequisite to war. In the next section each state is assumed to be uncertain of the other's willingness to use force.

Before turning to incomplete information and an analysis of the game's equilibrium, however, it will be useful to consider some of the limitations and qualifications of the model. The most significant limitation is that there are only two states. One argument about the relation between stability and the distribution of power emphasizes the importance of the number of coalitions that could form to block any state's bid for hegemony. The larger the number of states and the more even the distribution of power, the larger the number of potential blocking coalitions and the less likely is war.16 As a formal analysis of this argument clearly requires a model in which there are more than two actors, it is beyond the scope of the present analysis.17

It is important to emphasize, however, that although some arguments about the relation between the distribution of power and the probability of war presuppose the existence of more than two states, other arguments do not. Wright, for example, explicitly considers the situation in which there are only two states. In this case, balance-of-power theory expects that “there would be great instability unless [the two states] were very nearly equal in power or their frontiers were widely separated or difficult to pass.”18 Similarly, Mearsheimer considers the case of two great powers and claims that a balance of power is more stable: “Power can be more or less equally distributed among the major powers of both bipolar and multipolar systems. Both are more peaceful when equality is greatest among the poles.”19 In sum, the conflicting claims of the balance-of-power and preponderance-of-power schools apply to systems in which there are only two states, as well as to systems with more than two states. The present analysis focuses on the former case.

A second significant limitation of the model is that the distribution of power is fixed. But the key to several arguments about the relation

16 See Levy (fn. 1), 231–32; Mansfield (fn. 2, 1994), 17–18; and Wright (fn. 1), 755.
17 For efforts to examine the relation between stability and the distribution of power in a setting in which there are more than two states, see Emerson Niou and Peter Ordeshook, “Stability in Anarchic International Systems,” American Journal of Political Science 84 (December 1990); Wagner (fn. 1); and idem, “The Theory of Games and the Balance of Power,” World Politics 38 (July 1986).
18 Wright (fn. 1), 755.
19 Mearsheimer (fn. 1), 18; emphasis added.
between the likelihood of war and the distribution of power is that this
distribution shifts over time, perhaps because of uneven economic
growth. This raises an important problem, which is the effects of
changes in the distribution of power over time on the probability of
war. Unfortunately, it is beyond the scope of the present model and
must await future work.

Another seeming limitation is that war can result in only two possi-
ble outcomes: a state either wins all of the territory or loses all of it.
This limitation is more apparent than real, however. The assumption
that war can end in only two ways simplifies the notation and discus-
sion, but it is not central to the analysis of the model. One might as-
sume instead that war could result in any territorial division. Changes
in the distribution of power would still be reflected in changes in the
probability distribution over the now larger set of possible outcomes.

Furthermore, both states are assumed to agree on the distribution of
power (that is, both states know $p$). This assumption not only simplifies
the analysis but has some substantive import for international relations
theory as well. Blainey and others argue that war results from uncer-
tainty about the distribution of power. While this uncertainty may be
an important cause of war, it is not a necessary condition as Blainey
suggests. The model serves as a counterexample to this claim: states
agree on the distribution of power and yet there is war.

Finally, the terms of the agreement do not affect the distribution of
power. So, for example, an agreement that transfers a large amount of
territory to one state does not make that state more powerful. Wagner
argues that this may be an important cause of war, and extending the
model to allow for this possibility would be relatively straightforward.

The Equilibrium

This section characterizes the game’s equilibrium. To summarize the
results, a state will be called dissatisfied if it prefers fighting to the sta-
tus quo and satisfied if it prefers the status quo to fighting. If both states

20 See, for example, Charles Doran and Wes Parsons, “War and the Cycle of Relative Power,” Ameri-
can Political Science Review 74 (December 1980); Gilpin (fn. 5); Organski (fn. 1); and Organski and
Kugler (fn. 1).

21 For efforts in this direction, see Fearon (fn. 4); Woosang Kim and James Morrow, “When Do
Power Transitions Lead to War?” American Journal of Political Science 36 (November 1992); and Pow-
ell, “Appeasement as a Game of Timing” (Manuscript, Department of Political Science, University of
California, Berkeley, July 1995).

22 Blainey (fn. 1).

23 Fearon (fn. 4) first makes and develops this point.

24 Wagner (fn. 1).
are satisfied, then the status quo remains unchanged and the probability of war is zero, as neither state can credibly threaten to use force to overturn the status quo. If one of the states is dissatisfied, then the satisfied state makes its optimal offer to the dissatisfied state given the satisfied state's beliefs about the willingness of the dissatisfied state to use force. Although the dissatisfied state can always reject this offer and make a counteroffer, it never does so in equilibrium. Rather, the dissatisfied state either accepts this offer or attacks. The next section uses these equilibrium strategies to calculate the probability of war as a function of the distribution of power. (Readers less interested in the derivation of these results may omit the rest of this section.)

Three preliminaries are needed before the equilibrium strategies can be specified. First, a dissatisfied state must be defined more precisely. Second, the equilibrium of the complete-information game must be described. Finally, incomplete information needs to be introduced.

A state is dissatisfied if it prefers fighting to living with the status quo. In symbols, \( S_1 \)'s payoff to fighting now (that is, at \( t = 0 \)) is \( W_1(0) = (p - c_1)/(1 - \delta) \). \( S_1 \)'s payoff to living with the status quo is \( U_1(q)/(1 - \delta) \). Thus, \( S_1 \) is dissatisfied if \( p - c_1 > U_1(q) \). Similarly, \( S_2 \) is dissatisfied if \( 1 - p - c_2 > U_2(1 - q) \). At most only one state can be dissatisfied.

The complete information game has a very simple equilibrium. If both states are satisfied, then neither state can credibly threaten to use force to change the status quo. The status quo remains unaltered. If one of the states is dissatisfied, then the satisfied state offers just enough in equilibrium to the dissatisfied state to ensure that it will never find fighting worthwhile. In effect, the satisfied state appeases the dissatisfied state by offering it just enough to leave it indifferent between accepting the offer and fighting. In symbols, suppose that \( S_1 \) is the dissatisfied state. Then the satisfied state, \( S_2 \), will offer just enough, say \( x \), so that the dissatisfied state's payoff to accepting \( x \), \( U_1(x)/(1 - \delta) \), is equal to the payoff to fighting, \( (p - c_1)/(1 - \delta) \). Thus, \( x \) satisfies \( U_1(x) = p - c_1 \). To see that this is \( S_2 \)'s optimal offer, note that \( S_2 \) would never

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25 As will be seen, at most only one state can be dissatisfied.

26 This follows from the assumptions that the states are risk neutral or risk averse, that they agree on the distribution of power, and that fighting is costly. Because the states are risk neutral or risk averse, the utility functions are concave, which implies \( U_1(q) \geq q \) and \( U_2(1 - q) \geq 1 - q \). If both states are dissatisfied, then \( p - c_1 > U_1(q) \geq q \) and \( 1 - p - c_2 > U_2(1 - q) \geq 1 - q \). Adding these inequalities gives \( 1 - c_1 - c_2 > 0 \) or \( c_1 + c_2 < 1 \). But fighting is costly, so \( c_1 \geq 0 \) and \( c_2 \geq 0 \). Thus, \( c_1 + c_2 \) cannot be less than zero, and this contradiction implies that at least one state must be satisfied.

27 The complete-information game is a straightforward modification of Rubinstein's (fn. 12) model with an outside option. For an analysis of the unique subgame-perfect equilibrium of this game, see Martin Osborne and Ariel Rubinstein, *Bargaining and Markets* (New York: Academic Press, 1990), 54-58.
offer more than the minimum needed to appease $S_1$ and to ensure that it will not attack. Nor would $S_2$ offer less than this amount. $S_1$ would reject a smaller offer and fight, which would leave $S_2$ with a payoff of $(1 - p - c_2)/(1 - \delta)$. $S_2$, however, prefers the payoff of offering $x$ to $S_1$, which is $U_2(1 - x)/(1 - \delta)$.\textsuperscript{28}

With complete information, bargaining never results in war regardless of the distribution of power. The satisfied state always offers the dissatisfied state just enough to appease it. By contrast, bargaining can end in war if there is incomplete information, because the satisfied state is uncertain of what is needed to appease the dissatisfied state. The more the satisfied state offers, the less likely the dissatisfied state will be to attack; but the more the satisfied state offers, the lower its payoff will be should the offer be accepted. The satisfied state must therefore balance these two effects when deciding how much to offer. Often the satisfied state will choose to accept some risk of war. With incomplete information, the relation between stability and the distribution of power is a live issue.

To introduce incomplete information formally, each state will be assumed to be uncertain of the other state’s cost of fighting. In particular, $S_1$ is unsure of $S_2$’s cost of fighting $c_2$ but believes that this cost is at least $\xi_2$ and not more than $\xi_2$. The value $\xi_2$, which is assumed to be nonnegative, is the lowest cost of fighting that $S_1$ believes $S_2$ might have. That is, if $S_2$’s cost is $\xi_2$, $S_1$ is facing the toughest type of $S_2$. If $S_2$’s cost is $\xi_2$, $S_1$ is facing the weakest possible type of $S_2$; that is, the type of $S_2$ for which fighting is most costly. Similarly, $S_2$ is unsure of $S_1$’s cost of fighting but believes that it is at least $\xi_1$ and no more than $\xi_1$. These beliefs are represented formally as probability distributions $F_1(\xi_1)$ and $F_2(\xi_2)$.\textsuperscript{29}

Although $\xi_1$ and $\xi_2$ have been described as the states’ costs of fighting, these variables also have a more general interpretation. The lower $\xi_1$, the higher $S_1$’s payoff to fighting. Accordingly, $\xi_1$ can be interpreted more generally as a measure of $S_1$’s willingness to use force or of $S_1$’s resolve. That is, the lower $\xi_1$, the more willing $S_1$ is to use force and the greater its resolve. Thus, the game may be seen more broadly as a model of bargaining between two states that are unsure of each other’s willingness to use force.

The introduction of incomplete information necessitates a more refined definition of what it means to be dissatisfied. From $S_2$’s perspec-

\textsuperscript{28}If $S_2$ preferred fighting to offering $x$, then $1 - p - c_2 > U_2(1 - x)$ where $p - c_2 = U_1(x)$. Adding these relations and recalling that $U_1$ and $U_2$ are concave leave the contradiction $0 > \xi_1 + \xi_2$.

\textsuperscript{29}These cumulative distribution functions are assumed to have continuous densities that are positive over the intervals $(\xi_1, \xi_1)$ and $(\xi_2, \xi_2)$; these distributions are also common knowledge.
tive, $S_1$ can be any one of a continuum of types with costs ranging from $c_1$ to $c_2$. That is, $S_1$ is unsure of $S_1$'s type where $S_1$'s type is its cost of fighting. Similarly, $S_2$ is uncertain of $S_2$'s type where $S_2$'s type is its cost of fighting. Accordingly, a player type is dissatisfied if it prefers fighting to the status quo. So, type $c_1$ of $S_1$ is dissatisfied if $p - c_1 > U_1(q)$, and type $c_2$ of $S_2$ is dissatisfied if $1 - p - c_2 > U_2(1 - q)$. A player is potentially dissatisfied if there is some chance that it prefers fighting to the status quo. That is, a player is potentially dissatisfied if its toughest type is dissatisfied. Accordingly, $S_1$ is potentially dissatisfied if $p - c_1 > U_1(q)$, and $S_2$ is potentially dissatisfied if $1 - p - c_2 > U_2(1 - q)$. Paralleling the complete-information case, at most only one state can be potentially dissatisfied.

The incomplete-information game has a unique perfect Bayesian equilibrium outcome. If both states are satisfied, then the outcome is trivial. Although each state is uncertain of the other's exact cost of fighting, each is sure the other's cost is so high that the other state will not fight to overturn the status quo. Thus, neither state can credibly threaten to use force to change the status quo. In equilibrium, the status quo goes unchanged, and the probability of war is zero.

The derivation of the equilibrium in the case in which one of the states is potentially dissatisfied is long and very detailed and is presented in full detail elsewhere. The present discussion focuses on the central ideas underlying the proof. There are three major steps in the derivation. Assume without loss of generality that $S_1$ is the potentially dissatisfied state. Then the first step is to show that in equilibrium no dissatisfied type of $S_1$ will ever reject an offer from $S_2$ in order to make a counteroffer. A dissatisfied type will either accept the offer on the table or fight, depending on which alternative gives the higher payoff. Second, the fact that no dissatisfied type will ever make a counteroffer implies that all satisfied types will accept any offer larger than the status quo. The first two steps thus characterize the dissatisfied state's response to an offer from the satisfied state: If the satisfied state $S_2$ offers an $x$ that is larger than the dissatisfied state's status quo share of $q$, then

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30 Although the outcome is unique, there are multiple equilibria because different off-the-equilibrium-path beliefs will support this equilibrium. The fact that there is a unique outcome is surprising. Typically in bargaining games in which an informed bargainer (i.e., a bargainer with private information) can make offers, there is a multiplicity of equilibrium outcomes. For an excellent introduction to bargaining models, see Drew Fudenberg and Jean Tirole, Game Theory (Cambridge: MIT Press, 1991). A good survey is also found in John Kennan and Robert Wilson, "Bargaining with Private Information," Journal of Economic Literature 31 (March 1993).

all types of $S_1$ that prefer $x$ to fighting accept $x$ and all other types attack. Given this response, the final step in the derivation of the equilibrium is to describe the satisfied state's optimal offer given its cost of fighting, its beliefs about $S_1$'s cost of fighting, and $S_1$'s response.

Step 1. A dissatisfied type will never reject an offer in order to make a counteroffer. It will either accept the offer on the table or fight.

A preliminary to establishing this claim is to put upper bounds on the satisfied state's offers and acceptances. Let $S_2$ be the satisfied state and suppose that $S_2$ is deciding what to offer $S_1$ at any point in the bargaining game. $S_2$ will have updated its initial beliefs about $S_1$'s cost in light of the offers $S_1$ has previously made. That is, $S_2$ will have revised its initial beliefs about $S_1$'s cost, which are represented by the probability distribution $F_1$, in light of $S_1$'s previous actions. These updated beliefs can also be represented by a new probability distribution.

Let $s_1$ denote the toughest type of $S_1$ that $S_2$ believes it might be facing following a sequence of offers and counteroffers denoted by $h_t$. To put an upper bound on what the satisfied state might offer at any subsequent time, let $\zeta$ be $s_1$'s cost of fighting. If $s_1$ is a dissatisfied type, then the satisfied state $S_2$ will never offer more than what it takes to appease the toughest type that it might be facing. This is the smallest offer that ensures that $s_1$ will not attack. Any higher offer would also ensure that $S_2$ would not be attacked but would mean a lower payoff if the offer were accepted. Formally, if $s_1$ is dissatisfied, $S_2$ will never offer more than $x$ where $x$ satisfies $U_1(x) = p - \zeta$. If $s_1$ is satisfied, then $S_2$, although uncertain of the other state's exact cost of fighting, is sure that the other state is unwilling to use force to alter the status quo. In these circumstances, $S_2$ would never offer to revise the status quo in the other state's favor. In sum, $S_2$ would never offer more than the larger of $x$ and $q$ at any time after the initial sequence of offers and counteroffers $h_t$.

To put an upper bound on the demands to which $S_2$ might accede following $h_t$, two cases must also be considered. First, suppose that the toughest type that $S_2$ might be facing, $s_1$, is satisfied and therefore unwilling to use force to overturn the status quo. If in this case, $S_2$ will find any threat to use force incredible. Hence, the largest demand that $S_2$ might accept in this case is a demand of $q$, which is really a "demand" to ratify the status quo. (Because $S_2$ is satisfied, it is also unwilling to revise the status quo forcibly.)

In the second case, $s_1$ is dissatisfied. In these circumstances, if $S_2$ rejects the dissatisfied state's demand and counters with its maximal offer
of $\hat{x}$, this counter will be accepted immediately. Given the immediate acceptance of $\hat{x}$, $S_2$'s payoff to rejecting a demand in order to counter with $\hat{x}$ is the payoff to having $1 - q$ for one more period, that is, the period in which it rejects the offer, and then having the share $1 - \hat{x}$ forever. This payoff is $U_2(1 - q) + \delta U_2(1 - \hat{x})/(1 - \delta)$. Thus, $S_2$ would never agree to a demand from $S_1$ that left $S_2$ with a less than $U_2(1 - q) + \delta U_2(1 - \hat{x})/(1 - \delta)$, for $S_2$ could do better by rejecting the demand and countering with $\hat{x}$. Accordingly, the only demands $y$ which $S_2$ might accept must satisfy $U_2(1 - y)/(1 - \delta) \geq U_2(1 - q) + \delta U_2(1 - \hat{x})/(1 - \delta)$. Solving for $y$ gives $y \leq 1 - U_2^{-1}((1 - \delta)U_2(1 - q) + \delta U_2(1 - \hat{x}))$. $\hat{y}$, therefore, is an upper bound on the demands $\hat{s}_2$ might accept when it believes that the toughest type it might be facing is $\hat{s}_1$.

Summarizing, $S_2$ will never offer more than the larger of $q$ and $\hat{x}$. Nor will $S_2$ agree to any demand giving the dissatisfied state more than the larger of $q$ and $\hat{y}$.

These upper bounds imply that no dissatisfied type of $S_1$ will reject an offer to make a counteroffer. This claim will be established by arguing by contradiction. That is, a dissatisfied type of $S_1$ will be assumed to reject an offer in order to make a counteroffer, and this assumption will be shown to lead to a contradiction.

Suppose there is an equilibrium in which, following a sequence of offers and counteroffers $h$, which ends in an offer of, say, $x$ from $S_2$, a dissatisfied type of $S_1$ rejects $x$ in order to make a counteroffer of some $y$. Let $\hat{s}_1$ denote the toughest type of $S_1$ that $S_2$ believes it might be facing after receiving the demand $y$, and let $\hat{c}_1$ be $\hat{s}_1$'s cost of fighting. In equilibrium, $\hat{s}_1$ must have been willing to make the counteroffer of $y$; otherwise $S_2$ would not believe that there is some chance that it might be facing $\hat{s}_1$. But $\hat{s}_1$ would be willing to counter with $y$ only if there were some chance that doing so would bring it at least as much as it could have had by simply attacking.

When considering whether to attack or counter with $y$, $\hat{s}_1$'s payoff to attacking is $W_1(0) = (p - \hat{c}_1)/(1 - \delta)$. As will be seen, the upper bounds on $S_2$'s offers and acceptances show that the payoff to countering is always less than $W_1(0)$. The fact that $\hat{s}_1$ could have done better by attacking rather than countering contradicts the assumption that $\hat{s}_1$ rejects an offer in equilibrium in order to make a counteroffer, for no type can

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32 To see that this offer will be accepted immediately, recall that $\hat{x}$ is designed to ensure that the dissatisfied state cannot do better than accepting $\hat{x}$ by fighting instead. In equilibrium, the dissatisfied state will never reject $\hat{x}$ in order to fight. And since $S_2$ will never offer more than $\hat{x}$, the dissatisfied state can gain nothing by holding out for a better offer. Indeed, the dissatisfied state will lose by not reaping the benefits from a favorable shift in the status quo from $q$ to $\hat{x}$ as soon as possible. Thus, the dissatisfied state will accept a counteroffer of $\hat{x}$ immediately.
have a positive incentive to deviate from its equilibrium strategy. This contradiction will establish the claim made in step 1.

If \( s_1 \) rejects an offer to make a counteroffer, the game could end in only one of three ways following the counter. First, the game could end in war in some future period. But the payoff to a dissatisfied type to living with the status quo for a while and then fighting is strictly less than the payoff to fighting now. In symbols, \( W_1(t) < W_1(0) \) for \( t \geq 1 \). Thus, \( s_1 \) strictly prefers not to counter if the game is eventually going to end in war.

The second way that the game could end is that \( s_1 \) could ultimately accept an offer from \( S_2 \). But as shown above, \( S_2 \) never offers more than \( x \). Consequently, \( s_1 \)'s maximum payoff to countering with \( y \) if the game subsequently ends with \( s_1 \)'s acceptance of an offer of \( x \) is \( s_1 \)'s payoff to living with the status quo for two periods—namely, the period in which \( s_1 \) counters with \( y \) and then the period in which \( S_2 \) rejects this offer in order to counter with \( x \)—and then having \( x \) forever. In symbols, this payoff is \( U_1(q) + \delta U_1(y) + \delta^2 U_1(1 - x)/(1 - \delta) \). But this payoff is less than \( W_1(0) = (p - c_1)/(1 - \delta) \), because \( U_1(x) = p - c_1 \) and, since \( s_1 \) is dissatisfied, \( p - c_1 > U_1(q) \). Again, \( s_1 \) does strictly better by fighting rather than countering if the game ends with \( s_1 \)'s ultimately accepting an offer from \( S_2 \).

The final way that the game could end is with \( S_2 \)'s agreeing to a demand from \( s_1 \). \( s_1 \)'s maximum payoff to this outcome occurs if \( s_1 \) immediately counters with the maximal acceptable demand \( y \) and \( S_2 \) accepts. This would leave \( s_1 \) with the payoff to living with the status quo for the period in which it rejects \( x \) to counter with \( y \) and then having \( x \) forever. In symbols, \( s_1 \)'s payoff is \( U_1(q) + \delta U_1(y)/(1 - \delta) \). But this payoff is strictly less than \( s_1 \)'s payoff to fighting, \( (p - c_1)/(1 - \delta) \). As before, \( s_1 \) does strictly better by fighting rather than countering if the game ends with \( s_1 \)'s accepting \( s_1 \)'s demand.

In sum, \( s_1 \) can increase its payoff by attacking rather than making a counteroffer regardless of how the game ends following \( s_1 \)'s counter. Accordingly, \( s_1 \) would be able to increase its payoff by deviating from its equilibrium strategy if its equilibrium strategy were to reject an offer in order to make a counteroffer. This, however, is a contradiction because the definition of an equilibrium requires that no actor can improve its

\[ U_2(1 - y)/(1 - \delta) = U_2(1 - q) + \delta U_2(1 - x)/(1 - \delta) \]. But \( S_2 \) prefers the status quo share of \( 1 - q \) to what it will have after settling on either \( 1 - y \) or \( 1 - x \). That is, \( 1 - q > 1 - y \). Given \( 1 - q > 1 - y \), then the previous equality implies \( 1 - y > 1 - x \) or, equivalently, \( x > y \). Finally, the fact that \( x > y \) leaves \( W_1(0) = (p - c_1)/(1 - \delta) > U_1(q) + \delta U_1(y)/(1 - \delta) \).
payoff by deviating from its equilibrium strategy. This contradiction thus establishes the claim that no dissatisfied type can reject an offer in equilibrium in order to make a counteroffer. If, moreover, no dissatisfied type will reject an offer to make a counter, then a dissatisfied type must choose one of the two remaining alternatives. It will either accept the offer on the table or attack, depending on which alternative yields the higher payoff.

Step 2. All satisfied types accept any offer larger than the status quo.

A ssume as before that $S_1$ is the potentially dissatisfied state and that the satisfied state has offered to revise the status quo in favor of $S_1$. Moreover, $S_2$ has offered $S_1$ a share $x$ where $x > q$. As shown in step 1, a dissatisfied type of $S_1$ rejects $x$ if this type's payoff to fighting is higher than the payoff to accepting $x$ and accepts $x$ otherwise.

Now consider the decision facing a satisfied type of $S_1$, which is denoted by $S_1$. $S_1$ never attacks. To see that $S_1$ does not attack, note that the payoff to accepting $x$ is greater than the payoff to living with the status quo because $x > q$, and the payoff to living with the status quo is larger than the payoff to fighting because $S_1$ is satisfied. Thus, the payoff to accepting $x$ is greater than the payoff to fighting, and therefore $S_1$ will not attack.

Given that $S_1$ will not attack, its decision reduces to accepting $x$ or rejecting it in order to make a counteroffer. $S_1$ will accept $x$ because it offers the higher payoff. To calculate the payoff to rejecting $x$, recall that no dissatisfied type of $S_1$ will reject $x$ in order to make a counteroffer. If, therefore, $x$ is rejected, this rejection is in effect a signal to $S_2$ that it is facing only satisfied types of $S_1$. But as soon as $S_2$ becomes convinced that it is facing only satisfied types of $S_1$, it will find any threat from $S_1$ to use force to overturn the status quo inherently incredible. Thus, $S_2$ will never agree to revise the status quo in $S_1$'s favor. The status quo, therefore, will not be revised following the rejection of $x$. Hence, $S_1$'s payoff to rejecting $x$ is the payoff to living with the status quo. But $x > q$, so the payoff to agreeing to $x$ is higher than the payoff to living with the status quo. Accordingly, satisfied types of the potentially dissatisfied state will accept any offer $x > q$.

To see what satisfied types do if $S_2$ offers or, more aptly, demands a revision of the status quo in its favor, suppose $x < q$. As before, all dissatisfied types of $S_1$ will either accept $x$ or fight, depending on which al-

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34 The analysis of the case in which $x = q$ is completely analogous.
ternative offers the higher payoff. Indeed, in the case in which $S_2$ offers less than the status quo, all dissatisfied types fight. (By definition, all dissatisfied types prefer fighting to the status quo, and, with $x < q$, the payoff to accepting $x$ is even less than the payoff to the status quo.) Because all dissatisfied types reject $x$ and fight, $S_2$ will infer once again that it is facing only satisfied types of $S_1$ if $x$ is countered. And, as just seen, once $S_2$ becomes convinced that it is facing only satisfied types, the status quo will not be changed. Thus, the choice confronting a satisfied type when $x < q$ is to attack, accept the offer, or reject it and live with the status quo. Of the three, a satisfied type prefers living with the status quo. The status quo payoff is higher than the payoff to accepting $x$ because $x < q$ and higher than the payoff to fighting because this type is satisfied. In sum, a satisfied type rejects $x$ in equilibrium if $x < q$ and it receives the payoff to living with the status quo.

Step 3: Specifying the satisfied state's optimal offers.

The first two steps in the derivation of the game's equilibrium describe how the types of the potentially dissatisfied state respond to an offer from the satisfied state. The satisfied state, in turn, will make its best offer in light of these responses and its beliefs about the dissatisfied state's costs. Let $s_2$ denote the type of $S_2$ with cost $c_2$. Then given its beliefs about the potentially dissatisfied state's costs, $s_2$ can determine the probability that $S_1$ will reject an offer of $x$ and attack. (Assume for now that $x > q$.) To wit, the type of $S_1$ with cost $c_1$ will reject $x$ and attack if this type is dissatisfied and prefers fighting to accepting $x$. In symbols, this type fights if $p - c_1 > U_1(x)$ or, equivalently, $p - U_1(x) > c_1$. But, $S_2$'s beliefs about $S_1$'s costs can be used to calculate the probability that this cost is less than $p - U_1(x)$ and therefore that an offer of $x$ will lead to war. Let $R(x)$ denote the probability that $S_1$ will attack in response to $x$.

If a type of $S_1$ rejects $x > q$ and fights, $s_2$'s payoff will be $(1 - p - c_2)/(1 - \delta)$. If $x$ is accepted, its payoff is $U_2(1 - x)/(1 - \delta)$. (As shown above, $x$ will never be countered when $x > q$.) Thus, $s_2$'s expected payoff to offering $x$ is the payoff if $S_1$ attacks in reply to $x$ times the probability that $S_1$ attacks plus the payoff if $x$ is accepted times the probability that $x$ is accepted. Letting $P(x)$ denote this expected payoff leaves:

$$P(x) = \left(\frac{1 - p - c_2}{1 - \delta}\right) R(x) + \left(\frac{U_2(1 - x)}{1 - \delta}\right) (1 - R(x))$$
s\_2, therefore, offers the value of x that maximizes P(x) subject to the condition that x ≥ q.\textsuperscript{35}

It is now straightforward to use the results of steps 1, 2, and 3 to describe the unique perfect Bayesian equilibrium outcome of the bargaining game. Suppose that the game begins with the satisfied state's making the initial offer to the potentially dissatisfied state. The potentially dissatisfied state will respond to this initial offer as outlined above: dissatisfied types will accept it or fight, depending on which alternative yields the higher payoff, and satisfied types will accept if the offer is more than the status quo and will reject if it is less than the status quo. The satisfied state will then use its initial beliefs about the potentially dissatisfied state's costs to determine the probability that the potentially dissatisfied state will respond to a particular offer by attacking. And given these probabilities, the satisfied state will make its optimal offer, which is the value of x that maximizes P(x) for x ≥ q and where the probability that an initial offer of x is rejected is R(x) = F_1(p - U_1(x)).\textsuperscript{36}

The probability that the bargaining will end in war is, in equilibrium, simply the probability that the potentially dissatisfied state will reply to this optimal initial offer by attacking.\textsuperscript{37}

**STABILITY AND THE DISTRIBUTION OF POWER**

The equilibrium of the bargaining model makes it possible to calculate the probability of war as a function of the distribution of power. This section calculates this relation in a specific case and then compares these results to the expectations of the balance-of-power and preponderance-of-power schools and to other formal work. The section also offers a possible explanation for the equivocal results of empirical efforts to determine the relation between stability and the distribution of power and briefly considers some of the difficulties of estimating this relation empirically. Finally, the section discusses the relation between the probability of war and the distribution of power in more general cases. As in the specific case analyzed in detail here, the expectations of

\textsuperscript{35} The expression for P(x) was derived on the basis of the assumption that x > q. If x ≤ q, s\_2's expected payoff to offering x is P(q). Accordingly, s\_2 is indifferent to offering any x ≤ q and it will suffice to maximize P over the range x ≥ q.

\textsuperscript{36} s\_1 rejects x if p - c_1 > U_1(x) or p - U_1(x) > c_1. Given that c_1 is distributed according to the cumulative distribution function F_1, the probability that c_1 is less than p - U_1(x) is F_1(p - U_1(x)).

\textsuperscript{37} Although cumbersome to show, the probability that the bargaining will end in war is essentially the same regardless of whether the satisfied state or the potentially dissatisfied state makes the initial offer. See Powell (fn. 31) for the details.
both the balance-of-power and the preponderance-of-power schools fail to hold in these more general cases.

To simplify the analysis, assume that the states are risk neutral and that each believes the other’s cost of fighting is uniformly distributed between zero and one. Risk neutrality and the previous normalizations of the states’ utility functions imply that $S_1$’s and $S_2$’s utilities for the division $(x, 1-x)$ are simply $x$ and $1-x$, respectively.

The probability of war turns out to be a simple function of the disparity between the distribution of power and the status quo distribution of territory. More specifically, the absolute value of the difference between the distribution of power and the status quo distribution of territory, $|p - q|$, is a measure of the disparity between these two distributions. Letting $\Pi(p,q)$ denote the probability of war as a function of $p$ and $q$, the appendix then shows:

$$\Pi(p,q) = \begin{cases} |p - q| - |p - q|^2 & \text{if } |p - q| \leq 1/2 \\ 1/4 & \text{if } |p - q| \geq 1/2 \end{cases}$$

These results seem quite intuitive. When the distribution of power mirrors the status quo, that is, when $p = q$, neither state expects to gain by using force to revise the status quo. The probability of war in this case is zero. In symbols, $\Pi(p,q) = 0$ when $p = q$. As the disparity between the distribution of power and the status quo distribution grows, the probability of war rises until it levels off at $1/4$.\(^{38}\)

Figure 2 plots the relation between the probability of war and the distribution of power for three different values of the status quo distribution. $S_1$’s status quo share of territory is $1/3$ if $q = 1/3$; $1/2$ if $q = 1/2$; and $2/3$ if $q = 2/3$. These results clearly contradict the expectations of both the balance-of-power and the preponderance-of-power schools. As illustrated in Figure 3a, the preponderance-of-power school expects the probability of war to be smallest when $S_1$ or $S_2$ is predominant (that is, when $p$ is large or small) and to increase as the distribution of power becomes more even (that is, as $p$ approaches $1/2$). But in the case analyzed here, the probability of war is smallest at $p = q$ and generally largest when one of the states is preponderant.

\(^{38}\) The specific value of $1/4$ at which the function levels off is a result of the particular assumption that the states’ costs are uniformly distributed between the bounds of zero and one. Different assumptions about these bounds would cause the function to level off at a different value.
FIGURE 2

THE PROBABILITY OF WAR AND THE DISTRIBUTION OF POWER

$q = 1/3$

$q = 1/2$

$q = 2/3$
Figure 3
Expectations of the Two Schools

(a) Preponderance of Power

(b) Balance of Power
Figure 3b illustrates the balance-of-power school’s expectations. The probability of war is smallest when there is a roughly even distribution of power (p is approximately 1/2) and increases as the distribution of power becomes skewed. These expectations correspond to the formal results only in the special circumstance in which the status quo division is even (q = 1/2). When the status quo division is uneven, as in Figure 2a and 2c, the probability of war in the model is smallest when the distribution of power mirrors the uneven status quo division and not, as the balance-of-power school asserts, when the distribution is even.

The present analysis also complements and qualifies the results of other formal analyses of the relation between stability and the distribution of power. Bueno de Mesquita and Lalman, for example, study a model in which two states bargain about revising the status quo.39 In their game each state has the opportunity to make a single demand of the other state. After both states have made their demands, each state decides whether or not to use force to secure its demand. In one version of this game Bueno de Mesquita and Lalman assume that the size of each state’s demand is determined outside the model. That is, the amount each state demands is simply one of the exogenously specified initial conditions of the model. In this version of Bueno de Mesquita and Lalman’s model, the size of a state’s demand is not determined as part of a state’s equilibrium strategy. A state does not determine how much it demands by balancing the risk of war against the chance of agreeing on a more favorable settlement. The justification for assuming the states’ demands to be exogenous is that these demands are “determined by internal political rules, procedures, norms, and considerations and may or may not be attuned to foreign policy considerations.”40 With the size of the states’ demands determined outside the model, Bueno de Mesquita and Lalman find that “dissatisfaction with the status quo is entirely unrelated to the likelihood of war.”41

The present analysis focuses specifically on how a state balances the greater risk that a larger demand will lead to war against the higher payoff the state will obtain if this larger demand is accepted. This focus requires a model in which the states determine the sizes of their de-

40 Ibid., 41.
41 Ibid., 190. In a second version of their game, Bueno de Mesquita and Lalman let the state’s demands be determined as part of their equilibrium strategies. But this game has complete information. The combination of endogenous demands and complete information means that the probability of war is zero in their model—as well as in the model analyzed above—regardless of the distribution of power. Bueno de Mesquita and Lalman do not examine the case in which there are both endogenous demands and incomplete information.
mands as part of their equilibrium strategies. The game examined here satisfies this requirement. The probability of war in the present game, unlike in Bueno de Mesquita and Lalman's model, is closely related to the level of dissatisfaction with the status quo. The larger the level of dissatisfaction in the case above, that is, the larger the disparity $|p - q|$, the more likely war.

Fearon also considers the relation between the probability of war and the distribution of power. He studies a game in which one state makes a take-it-or-leave-it offer after which a second state must then either accept this offer or attack. By assumption, this second state cannot make a counteroffer. The size of the state's demand in Fearon's model is determined as part of the state's equilibrium strategy, and he finds that the probability of war is "completely insensitive" to the distribution of power.

The current model complements Fearon's formulation. He is studying a stylized situation in which one state presents another state with an ultimatum in the form of a military fait accompli. This fait accompli establishes a new distribution that will become the new international status quo unless the second state goes to war to reverse it. In the stylized situation modeled in this essay, two states are bargaining about revising the status quo and have the opportunity to make offers and counteroffers. There are, however, no military faits accomplis: any use of force to revise the status quo is assumed to lead directly to war. In these circumstances the probability of war is sensitive to the distribution of power and, in particular, to the disparity between the distribution of power and the status quo distribution of territory.

The present analysis provides a possible explanation of the equivocal findings of empirical efforts to determine the relation between stability and the distribution of power. Empirical research has yielded at least three different findings about an even distribution: that it is more stable than a preponderance of power, that it is less stable than a preponderance of power, and, depending on whether one looked at the nineteenth or the twentieth century, that it is either more or less stable.

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42 Bueno de Mesquita and Lalman (fn. 2, 1992), 190.
43 Strictly speaking, the probability of war increases as $|p - q|$ increases only as long as $|p - q| < 1/2$.
44 After this, the probability of war levels off.
45 Fearon (fn. 4), 20.
46 Siverson and Tennfoss (fn. 2).
47 Kim (fn. 2, 1992); and Moul (fn. 2).
48 Singer, Bremer, and Stuckey (fn. 2).
Other work has found no significant relation between stability and the distribution of power.49

The equilibrium of the bargaining game shows that the probability of war depends on both the distribution of power and the status quo distribution. Thus, any attempt to assess the relation between the probability of war and the distribution of power should control for the status quo. Failing to do so will generally lead to biased estimates if, as might be expected, the distribution of power and the status quo distribution are correlated. But empirical efforts to estimate this relation have generally not controlled for the status quo, and this omission may partially account for the equivocal empirical results.50

Finally, it is important to ask about the generality of these results. Do the findings hold only in the specific example considered here in which the states were assumed to be risk neutral and to believe that each other’s cost or willingness to use force was uniformly distributed? Or do the results also hold in more general conditions? The central finding that the probability of war is at a minimum when the distribution of power mirrors the status quo distribution is quite robust. It holds as long as the states are risk neutral or risk averse, regardless of the particular shapes of their utility functions or of the particular shapes of the probability distributions representing their beliefs.51 Thus, the model contradicts the expectations of the balance-of-power and preponderance-of-power schools as long as the states are risk neutral or risk averse.52

One might also wonder if this result holds if the uncertainty sur-

49 Maoz (fn. 2); and Bueno de Mesquita and Lalman (fn. 2).

50 Unfortunately, the theoretical results derived here, while indicating the importance of controlling for the status quo, do not identify a means of doing so. The basic problem is to find a way of measuring a state’s utility for the status quo. Bueno de Mesquita and Lalman’s (fn. 2, 1992) measure of this utility may offer a start in this direction, but their current formulation is inadequate. If the states are risk neutral as in the example above, then Bueno de Mesquita and Lalman’s measure reduces to assuming that the status quo distribution is constant in all cases. In terms of the example above, they are in effect assuming that q always equals 1/2. (To obtain q = 1/2, assume the state is risk neutral by taking rA = 1 in equation A.13 in Bueno de Mesquita and Lalman (fn. 2, 1992), 294, and normalize the state’s utility to be one if it obtains everything it demands and zero if the other state obtains everything it demands by setting U1(q) = 1 and U2(1 – q) = 0.) Controlling for the status quo, however, requires the status quo to be treated as a variable across cases, and it is unclear how to do this with their measure as it is currently formulated.

51 To see this, suppose that the distribution of power closely mirrors the status quo distribution of territory in that |p – q| is less than the minimum of the lowest possible costs of fighting, i.e., the minimum of q and s. Then p – q ≤ s, and q – p ≤ s. These inequalities and the concavity of U1 and U2 imply 1 – p ≤ q ≤ U1(q) and 1 – q ≤ p ≤ U2(1 – q). Thus, both S1 and S2 are satisfied, and the probability of war is zero.

52 The case of risk-acceptant states introduces nonconvexities and technical difficulties, and the analysis of this case remains a task for future work.
rounding the states' willingness to use force varies with shifts in the distribution of power. More specifically, the costs of fighting in the example examined here are always uniformly distributed between 0 and 1 regardless of the distribution of power. But what would happen if the expected cost and uncertainty about a state's willingness to use force declined as that state became more powerful?53

This question requires a two-part response. First, the question itself takes the analysis beyond the balance-of-power and predominance-of-power schools, as neither school bases its argument on the subtleties implicit in this question. Second, the model provides a partial answer to this question. The probability of war is still zero as long as the status quo distribution of benefits mirrors the distribution of power.54 Once again, the expectations of the balance-of-power and preponderance-of-power schools fail to hold.55

**Conclusion**

The relation between the distribution of power and the probability of war is an important and long-debated problem in international relations theory. The balance-of-power school argues that an even distribution of power brings greater stability, while the preponderance-of-power school argues that a preponderance brings greater stability. These opposing claims have been examined in the context of an infinite-horizon bargaining game in which two states alternate making offers about how to revise the international status quo. The bargaining continues until the states agree on a revision or until one of them becomes sufficiently pessimistic about the prospects of reaching a mutually agreeable settlement that it resorts to force in an attempt to impose a new settlement. Contradicting the expectations of both the balance-of-power and preponderance-of-power schools, the probability of war in the model is smallest when the distribution of power mirrors the status quo distribution.

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53 More formally, suppose that the distributions of cost \( F_1 \) and \( F_2 \) were also a function of \( p \) and that the mean and variance of \( F_1 \) decrease in \( p \) while the mean and variance of \( F_2 \) decrease in \( 1 - p \).

54 Even if \( F_1 \) and \( F_2 \) depend on \( p \), the argument in footnote 26 goes through with only minor modification.

55 The model's answer is only partial because the shape of the function relating the probability of war to the distribution of power, while always zero when \( p = q \), is likely to depend on the states' utility functions and on the functional forms of the probability distributions representing their beliefs.
There are two cases to consider in calculating the probability of war as a function of the distribution of power. Suppose, first, that $p > q$. Then $S_1$ is the potentially dissatisfied state, because the expected payoff to fighting for the type of $S_1$ with the lowest cost of fighting is strictly greater than its status quo payoff: $p - c_1 = p - 0 > q$. Then, as shown above, the probability of war in the bargaining game is the probability that the dissatisfied state attacks in response to the satisfied state's optimal initial offer.

To calculate this probability, $S_2$'s optimal initial offer must first be determined. Suppose $S_2$ offers $x$ to $S_1$. A dissatisfied type $s_1$ of $S_1$ with cost $c_1$ will reject this offer and attack if its payoff to fighting is higher than its payoff to accepting. That is, $s_1$ attacks if $p - c_1 > x$ or, equivalently, if $p - x > c_1$. Thus, the probability that $S_1$ will attack in response to an offer of $x$ is the probability that $S_1$'s cost is less than $p - x$. Given that $S_1$'s costs of fighting are uniformly distributed between 0 and 1, the probability that $S_1$'s cost is less than $p - x$ is just $p - x$. Consequently, $S_2$'s expected payoff to offering $x$, which will be denoted by $P(x)$, is its payoff to fighting times the probability that $S_1$ will attack plus the payoff if $S_1$ accepts $x$ times the probability that $S_1$ will accept $x$. In symbols,

$$P(x) = \frac{1 - p - c_2}{1 - \delta} (p - x) + \frac{(1 - x)}{1 - \delta} (1 - (p - x))$$

if $S_2$'s offer is between $q$ and $p$. If $S_2$ offers more than $p$, then even the dissatisfied state with the lowest cost of fighting will prefer to accept the offer to fighting. Consequently, the probability that an offer of $x > p$ will be rejected is 0. $S_2$'s payoff to offering $x$ in this case reduces to $(1 - x)/(1 - \delta)$. If $S_2$ offers $x < q$, then all dissatisfied types fight and all other types reject the demand which leave the status quo unchanged. $S_2$'s payoff is:

$$\frac{1 - p - c_2}{1 - \delta} (p - q) + \frac{(1 - q)}{1 - \delta} (1 - (p - q)).$$

$S_2$'s optimal offer is the value of $x$ that maximizes its expected payoff. To describe the solution to this maximization problem, set the derivative of $P(x)$ equal to zero and let $x^*(c_2)$ denote the solution to this equa-
tion. This leaves $x^*_1(c_2) = p - \frac{(1 - c_2)}{2}$. Then the optimal offer of the type of $S_2$ with cost $c_2$ is $x^*_2(c_2)$ as long as $x^*_2(c_2) > q$. $S_2$'s optimal offer is $q$ if $x^*_2(c_2) < q$.

Now that the equilibrium offers have been specified, the probability of breakdown can be calculated. Let $\pi(c_2)$ be the probability that the type of $S_2$ with cost $c_2$ will be attacked. If $x^*_2(c_2) < q$ for this type, then this type will offer $q$. The probability that the dissatisfied state will attack in response to this offer is the probability that the cost of fighting $c_1$ is less than $p - q$ which, given the uniform distribution of costs, is simply $p - q$. Thus, $\pi(c_2) = p - q$ for all types of $S_2$ for which $x^*_2(c_2) < q$ or, equivalently, for all types of $S_2$ for which $c_2 < 1 - 2(p - q)$.

If $x^*_2(c_2) \geq q$ for a type of $S_2$, then this type will offer $x^*_2(c_2)$. The probability that the dissatisfied state attacks in response to this offer is $p - x^*_2(c_2) = p - \frac{(p - (1 - c_2)}{2} = \frac{(1 - c_2)}{2}$. Thus, $\pi(c_2) = \frac{(1 - c_2)}{2}$ whenever $x^*_2(c_2) \geq q$ or $c_2 \geq 1 - 2(p - q)$.

$\pi(c_2)$ specifies the probability that the particular type of $S_2$ with cost $c_2$ will fight. But from the perspective of an outside observer, as well as from that of $S_1$, $S_2$'s cost is uncertain. Thus the probability of war is the expected value of $\pi(c_2)$. Letting $\Pi(p,q)$ denote this expected value, then:

$$\Pi(p,q) = \int_0^1 \pi(c_2) dc_2$$

$$\Pi(p,q) = \begin{cases} (p - q) - (p - q)^2 & \text{if } 0 \leq p - q \leq 1/2 \\ 1/4 & \text{if } 1/2 \leq p - q \end{cases}$$

$\Pi(p,q)$ describes the relation between the probability of breakdown and the distribution of power if $p > q$ and, consequently, $S_1$ is the dissatisfied state. If $p < q$, then $S_2$ is the dissatisfied state and the relation between stability and the probability of war must be determined anew. Fortunately, the symmetry of the problem makes it easy to find this relation in this second case. Let $p' = 1 - p$ and $q' = 1 - q$. Then $p < q$ implies $p' > q'$. Moreover, $p'$ now measures the probability that the dissatisfied state will prevail and $q'$ measures the dissatisfied state's status quo payoff just as $p$ and $q$ did in the first case where $p > q$. The symmetry of the example then ensures that the probability of breakdown if $p <
q is given by $\Pi(p', q')$. But $\Pi(p', q')$ is just $\Pi(1 - p, 1 - q)$ which in turn equals $(q - p)(1 - (q - p))$ if $1 - 2(q - p) \geq 0$ and $1/4$ if $1 - 2(p - q) < 0$. Combining these results leaves:

$$\Pi(p, q) = \begin{cases} |p - q| - (p - q)^2 & \text{if } |p - q| \leq 1/2 \\ 1/4 & \text{if } |p - q| \geq 1/2 \end{cases}$$