REVIEW OF GAMES OF COMPLETE INFORMATION

Show all steps and calculations in your answers, for proofs justify each step.

QUESTION 1. Some bullets are loaded into a six-shooter. The cylinder is spun and the revolver is pointed at your head. Assuming that you prefer life to death and more money to less (if you die, you don't care how much money you paid), would you be willing to pay more to have one bullet removed when only one bullet was loaded or when three bullets were loaded? Can your preferences be represented by an expected utility function? If not, would you be willing to amend your statement after having worked through the logic?

QUESTION 2. Show that the two-player game in Figure 1 has a unique equilibrium. (Hint: Show that it has a unique pure strategy equilibrium. Then show that player 1 cannot put positive weight on any combination of pure strategies.)

		Player 2		
		L	C	R
	U	$1, -2$	$-2, 1$	$0, 0$
Player 1	M	$-2, 1$	$1, -2$	$0, 0$
	D	$0, 0$	$0, 0$	$1, 1$

Figure 1: The Unique Equilibrium Game.

QUESTION 3. Two students sign up to prepare an honors thesis with a professor. Each can invest time in his own project: either no time, one week, or two weeks (these are the only three options). The cost of time is 1 unit of payoff for each week spent on the project. The more time a student puts in, the better the work, and if they put in the same amount of time, the quality is the same. The professor will give an A to the higher quality work, a B to the lower quality work, and will toss a fair coin to decide which one gets the A and which one gets the B if the quality is the same. To each student, A is worth 3 units of payoff, and a B is worth 0. Answer each of the following:

(a) Write the game in strategic form.

(b) Are there any strictly dominated strategies? Are there any weekly dominated strategies?

(c) Find all Nash equilibria. Discuss the meaning of the results.

QUESTION 4. THE TRAGEDY OF THE COMMONS. There are $N \geq 2$ players who wish to enjoy a pretty shoreline of a mountain lake. The shoreline has recently been opened to privatization and players simultaneously decide whether to appropriate access by spending effort $e \in [0, 1]$ on such activities. (That is, $e = 0$ means that the player abstains from attempts to privatize access, and $e = 1$ means that he has privatized everything that a single player can privatize.) The payoff for a player is given by $u_i = 1 - 3\overline{e} + 2e_i$, where $\overline{e}$ is the average effort of the population of N players. Find the PSNE for this game. Interpret the result by comparing the equilibrium payoff to the situation where the shoreline is not open to privatization.

QUESTION 5. THE PIRATES AND THE SPOILS. There are n pirates, all ranked according to a strict hierarchy with the first pirate being the Captain, the second being the next in command, and so on and so forth down to the very last pitiful crewman. For simplicity, assume pirate 1 is the captain, 2 is the next in command, and so on down to n, who is the lowly crewman.

Presently, the pirates capture a merchant ship and have to decide how to divide the spoils consisting of $m > n$ gold coins. The procedure they use is as follows. The highest ranking pirate proposes some distribution of the coins. If at least half of the pirates agree to the proposal, it is implemented and the game ends. If, however, more than half the pirates fail to agree, then the proposer is made to walk the plank to his death and the next highest ranking pirate becomes the proposer. The procedure is then repeated. Pirates are self-preserving, greedy, and vindictive in that order: they most prefer to live, then to get as much gold coins as possible, and given both of these, to see as many superiors walk the plank as possible.

What is your guess about the Captain's predicament at the outset: is he in a strong position or is he screwed? (Write this down *before* solving the game.) What is the subgame perfect equilibrium of this game? How does this correspond to your prediction?

QUESTION 6. GIBBONS (1988). Consider the following game with three players. Player 1 chooses A or B, and if he chooses A, the game ends with payoffs $(6, 0, 6)$. If he chooses B, player 2 chooses C or D, and if she chooses C, the game ends with payoffs $(8, 6, 8)$. If she chooses D, players 1 and 3 play the following simultaneous-move coordination game:

		Player 3: E	F
Player 1	G	7, 10, 7	0, 0, 0
	H	0, 0, 0	7, 10, 7

(a) Draw the extensive-form representation of this game.

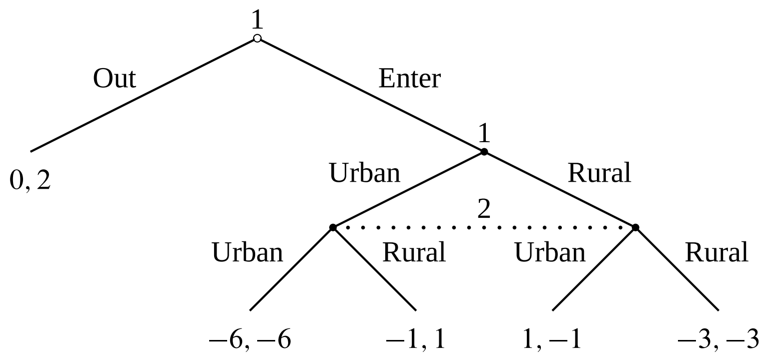
(b) Prove that in every subgame perfect equilibrium, player 1 chooses B at the outset.

(c) Define a hypothetical situation, in which players 1 and 2 both predict Nash equilibria in subgames, and doing so rationalizes player 1 choosing A at the outset. Why is this *not* an SPE?

(d) Find a Nash equilibrium in which player 1 chooses A at the outset. Why is this *not* an SPE?

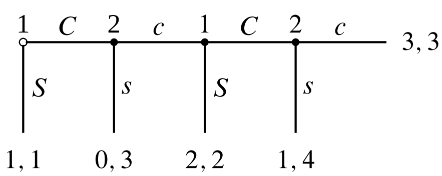
QUESTION 7. Consider the following incumbent-challenger game, in which once a challenger enters, both candidates must decide to campaign in rural areas or urban areas.

Find all Nash equilibria. Which ones are subgame perfect?



QUESTION 8. There are two players, a buyer and a seller. The buyer's value for the object is $v > 0$. Initially, the buyer chooses an investment level I that can be either high, I_H, or low, I_L, with $I_H > I_L$. This increases the buyer's value of the object to $v + I$ but costs I^2. The seller does not observe the investment level and offers the object at a price p. If the buyer accepts, his payoff is $v + I - p - I^2$, and the seller's payoff is p. If the buyer rejects, his payoff is $-I^2$, and the seller's payoff is 0. Find the subgame perfect equilibria.

QUESTION 9. Two players play two games sequentially. They observe the outcome from the first game, and their payoffs are the time-discounted payoffs from each of the games. That is, let u_i^n be player i's payoff from game n. Player i's total payoff is then $u_i^1 + \delta u_i^2$ where $\delta \in [0, 1]$ is the common discount factor. The games are given in Figure 2.



	L	R
U	1, 1	0, 0
D	0, 0	3, 3

Figure 2: The Two Stage Games.

(a) What are the Nash equilibria of each stage game?

(b) How many pure strategies does each player have in the multistage game?

(c) Find all pure-strategy subgame-perfect equilibria when $\delta = 0$.

(d) Find a subgame-perfect equilibrium for the multistage game in which players receive the payoffs $(2, 2)$ in the first stage when $\delta = 1$.

(e) What is the smallest discount factor that can support the subgame-perfect equilibrium you found in (d)?

(f) For values of δ greater than the one you found in (e), are there other outcomes of the first-stage game that can be supported in a subgame-perfect equilibrium?