REPEATED GAMES AND BARGAINING

Show all steps and calculations in your answers, for proofs justify each step.
Questions marked with '' are harder and result in extra points.*

QUESTION 1. Consider the Prisoner's Dilemma stage game shown in Figure 1 that is repeated infinitely, with players sharing a common discount factor $\delta \in (0, 1)$.

	C	D
C	$2, 2$	$0, 3$
D	$3, 0$	$1, 1$

Figure 1: Prisoner's Dilemma.

Can the following pairs of strategies be supported in SPE?

(a) Pavlov (win-stay, lose-shit) is a strategy that chooses the same action if the last outcome was relatively good and switches the action if the last outcome was relatively bad. The strategy starts by cooperating and then cooperates if the last outcome was either (C, C) or (D, D), and defects otherwise.

(b) Dev1l (deviate once) is a strategy that attempts to exploit the opponent just once, if at all possible. It prescribes playing Tit-for-Tat (TFT) until period T, defecting in period T, cooperating in $T + 1$, and playing TFT thereafter.

(c) Grim Dev1l is a strategy that attempts to be "nice" until some pre-specified period, then preempts by defecting first and never resumes cooperation afterwards. It prescribes playing TFT until period T, and then defecting forever starting in period T.

QUESTION 2. Two firms are located adjacent to one another on a lake, and each imposes an external cost on the other. The detergent that firm 1 uses in its laundry business makes the fish that firm 2 catches in the lake taste funny, while smoke that firm 2 emits in its fish processing makes the clothes that firm 1 is drying smell funny. Each firms profits are therefore increasing in its own production but decreasing in the production of the other firm. Let $q_i \geq 0$ be the per-period (stage game) production level of firm i so that its profit is $v(q_i, q_{-i}) = (30 - q_{-i})q_i - q_i^2$.

(a) Draw the firms' best-response functions for the stage game and find its Nash equilibrium. How does this compare to the Pareto-optimal stage-game production levels?

(b) Find an SPE in the infinitely repeated game with common discount factor $\delta \in (0, 1)$ that will allow the firms to achieve the Pareto-optimal production levels.

QUESTION 3. Consider an infinitely repeated game between two players with a common discount factor $\delta \in (0, 1)$. In each period, players play the stage game in Figure 2, in which player 1 chooses whether to trust player 2 (T) or not (N), and if he trusts her, player 2 chooses whether to cooperate (C) or defect (D).

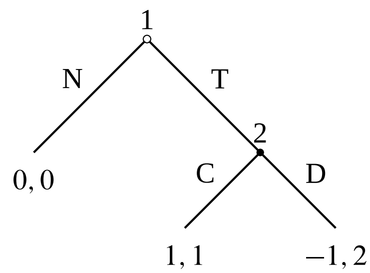

Figure 2: Trust Stage Game.

(a) Draw the convex hull of average payoffs in the repeated game and indicate which of these are also individually rational.

(b) Find the SPE with non-contingent strategies.

(c) Find the SPE that can support trust and cooperation using Grim Trigger strategies.

(d) Is there an SPE that can support the average payoffs $(-0.4, 1.1)$ for a large enough discount factor? If yes, show the strategies. If not, explain why not.

(e) (*) Construct an SPE that can support average payoffs that approach $(^1/_3, {}^4/_3)$ as $\delta \to 1$. (Hint: use a public randomization device so that players can coordinate on some pure-strategy play from which deviations can be easily detected.)

QUESTION 4. Consider an alternating-offers bargaining game in which the two players have to split a benefit of size 1, with player 1 making the first proposal. The players have different discount factors, δ_1 and δ_2. Each gets a payoff of 0 if no agreement is reached.

(a) Suppose that the game is played for only 5 periods. What is the outcome in the unique SPE?

(b) Are there conditions on the discount factors that give player 2 an advantage? Why?

(c) (*) Suppose now that every time players respond to an offer, they have the option to break off negotiations, which ends the game immediately with payoffs (w_1, w_2), where $w_i \in (0, 1)$. What is the outcome in the unique SPE? How does this differ from the model without this "outside option"?

(d) Are there conditions on the outside options that give player 2 an advantage? Why?

(e) Suppose now that the game without outside options is played over an infinite horizon. Find the unique SPE using the techniques we developed in class.

(f) What happens to the equilibrium payoffs if we increase the difference between the discount factors such that player 2 becomes more patient?