

The Principle of Convergence in Wartime Negotiations: A Correction

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Abstract. This note corrects Proposition 2 in Slantchev (2003).

1 Introduction

In Slantchev (2003), I claimed with Proposition 2 the existence of a Markov Perfect Sequential Equilibrium (henceforth, “equilibrium”) in which the delay in reaching an agreement is just long enough to reveal the private information player 2 has about the probability of winning battles. The proof proceeds by constructing this equilibrium in period $t = 2$, and then inducting backwards on both beliefs and strategies to establish the conditions that would sustain this behavior in periods $t = 0, 1$ as well. The mistake occurs in the calculation of player 1’s belief in $t = 1$ following 2’s equilibrium offer. The appendix correctly states that player 1’s ex ante expectation of winning a fight if it rejects the offer to be $p_1 = q_1^s p_L + (1 - q_1^s) p_M$, where q_1^s is player 1’s ex ante belief that it is facing the strong player 2, 2_s . However, in the proposed equilibrium, observing an acceptable offer by player 2 should lead player 1 to revise his beliefs such that $q_1^s = 0$ because only 2_m is actually willing to make such an offer. Hence, in the proposed equilibrium, player 1’s belief that he will win a battle following an acceptable offer is simply p_M because conditional on receiving such an offer, he will be certain that it is facing 2_m . Hence, equation (2), which states the value of that acceptable offer, must read:

$$y^*(k_1) = \pi - (1 - \delta)b_1 - \delta [p_M V_m^1(k_1 + 1) + (1 - p_M) V_m^1(k_1 - 1)].$$

This invalidates the rest of the proof and requires a different line of attack in solving for the equilibrium. Incidentally, the problem is endemic to the solution concept (perfect sequential equilibrium) itself, which may fail to exist if players are too patient. I provide more details in the section on signaling later on.

*I thank Yoji Sekiya for alerting me to the mistake.

2 Equilibrium when $q^s = 1$

This is identical to the original proof, and is provided here for completeness (the published article omits this for reasons of space). I now specify the strategies if 1 ever becomes convinced that he's facing 2_s ; that is, if $q^s = 1$. In this case, the belief will never get revised regardless of battle outcomes. I will also assume that it stays unchanged if players take actions inconsistent with their equilibrium strategies.

	$q^s = 1$
1 proposes	$x = V_s^1(k)$
1 accepts	$\pi - y \geq (1 - \delta)b_1 + \delta [p_L V_s^1(k + 1) + (1 - p_L)V_s^1(k - 1)]$
2_w proposes	$y = V_s^2(k)$
2_w accepts	$\pi - x \geq (1 - \delta)b_2 + \delta [\pi - p_H V_s^2(k + 1) - (1 - p_H)V_s^2(k - 1)]$
2_m proposes	$y = V_s^2(k)$
2_m accepts	$\pi - x \geq (1 - \delta)b_2 + \delta [\pi - p_M V_s^2(k + 1) - (1 - p_M)V_s^2(k - 1)]$
2_s proposes	$y = V_s^2(k)$
2_s accepts	$\pi - x \geq (1 - \delta)b_2 + \delta [\pi - p_L V_s^2(k + 1) - (1 - p_L)V_s^2(k - 1)]$
transitions	stay at $q^s = 1$ if anyone deviates from the equilibrium strategies regardless of the outcomes of battles

In equilibrium, the player whose turn is it to move will make the complete-information MPE proposal, and the other player will accept it. Note that even though 2_w and 2_m in principle would accept worse offers than 2_s (because their continuation values from rejection are lower), 1 will never make such offers in equilibrium: he believes that he's facing 2_s with certainty, so he believes that any deviation from the minimum acceptable offer for 2_s is sure to be rejected, and this delay is worse than satisfying 2_s . An analogous argument leads him to accept 2_s 's offer, which means that neither of the weaker types has any incentive to deviate: offering more leads to acceptance but lower payoff, and offering less leads to rejection and a delay which is also worse (they are more likely to end in a less attractive state because they are more likely to lose the battle). Hence, if player 1 ever becomes convinced that $q^s = 1$, the game will end immediately in the current state with the full-information MPE payoffs.

3 Equilibrium when $0 < q^s < 1$ and $q^m = 1 - q^s$

Suppose now that $0 < q^s < 1$ and $q_w = 0$; that is, player 1 is certain he is not facing 2_w but is unsure if he's facing 2_s or 2_m . I now specify the strategies such that, along the equilibrium path player 1 makes (belief-state stationary) offers that only 2_m accepts or else offers $V_s^1(k)$ that both types accept.

3.1 Player 1 Screens in Proposal Stage

Consider some period in which it is player 1's turn to make an offer when his prior belief is $0 < q^s < 1$, and $q^m = 1 - q^s$. He proposes $x(k)$ that only 2_m accepts and 2_s rejects. Since rejection signals unambiguously player 2's type, it follows that $q^s = 1$ should player

2 reject the equilibrium offer. Since 2_m must be willing to accept x and she can always mimic 2_s by rejecting it and because 1 will never offer more than the absolute minimum necessary to achieve that acceptance, it follows that:

$$\pi - x(k) = (1 - \delta)b_2 + \delta [p_M V_s^2(k + 1) + (1 - p_M)V_s^2(k - 1)], \quad (1)$$

where the continuation value follows from the strategies specified above for the case of $q^s = 1$. On the other hand, since 2_s is willing to reject, it follows that:

$$\pi - x(k) < (1 - \delta)b_2 + \delta [p_L V_s^2(k + 1) + (1 - p_L)V_s^2(k - 1)].$$

Finally, it must be the case that player 1 is willing to separate rather than make an offer that both types would accept. Since satisfying 2_s now requires offering her the full-information equivalent in the current period, it follows that player 1 will screen if:

$$(1 - q^s)x(k) + q^s [(1 - \delta)b_1 + \delta (\pi - p_L V_s^2(k + 1) - (1 - p_L)V_s^2(k - 1))] \geq V_s^1(k).$$

Noting now that

$$\pi - V_s^1(k) = (1 - \delta)b_2 + \delta [p_L V_s^2(k + 1) + (1 - p_L)V_s^2(k - 1)],$$

we can rewrite the last inequality as:

$$(1 - q^s)x(k) + q^s [(1 - \delta)b_1 + \delta(\pi - z)] \geq \pi - (1 - \delta)b_2 - \delta z(k),$$

where $z(k) \equiv p_L V_s^2(k + 1) + (1 - p_L)V_s^2(k - 1)$. Using the definition of $x(k)$ and letting $\hat{z}(k) \equiv p_M V_s^2(k + 1) + (1 - p_M)V_s^2(k - 1)$, we can rewrite the inequality again as:

$$\begin{aligned} (1 - q^s) [\pi - (1 - \delta)b_2 - \delta \hat{z}(k)] + q^s [(1 - \delta)b_1 + \delta(\pi - z(k))] \\ \geq (1 - \delta)b_1 + \delta(\pi - z(k)) + (1 - \delta)(\pi - b_1 - b_2), \end{aligned}$$

where we added and subtracted $(1 - \delta)b_1$ and $\delta\pi$ to the right-hand side. This now yields:

$$(1 - q^s) [(1 - \delta)(\pi - b_1 - b_2) + \delta(z(k) - \hat{z}(k))] \geq (1 - \delta)(\pi - b_1 - b_2),$$

or simply:

$$\left(\frac{\delta}{1 - \delta} \right) \left(\frac{z(k) - \hat{z}(k)}{\pi - b_1 - b_2} \right) \geq \frac{q^s}{1 - q^s},$$

where we used the fact that $\pi - b_1 - b_2 > 0$. Since the right-hand side is positive, for a solution to exist it must be the case that $z(k) - \hat{z}(k) > 0$, which I now show. Using the definitions, this difference yields:

$$(p_M - p_L) [V_s^2(k - 1) - V_s^2(k + 1)] > 0,$$

where the inequality follows from $p_M > p_L$ and $V_s^2(k - 1) > V_s^2(k + 1)$ (recall that higher states are worse for player 2). Intuitively, the difference $z(k) - \hat{z}(k)$ captures player 1's bonus from delaying agreement one period in an attempt to screen out the weaker player. As the expression above makes clear, it is the difference in the payoffs he would have to

offer after winning and losing the battle, times the difference in the probability of winning if he happens to fight the weaker type. This now gives us the condition necessary and sufficient to sustain player 1's screening strategy: $\frac{q^s}{1-q^s} \leq \zeta$, where for simplicity and compactness we let $\zeta(k) \equiv \left(\frac{\delta}{1-\delta}\right) \left[\frac{(p_M - p_L)(V_s^2(k-1) - V_s^2(k+1))}{\pi - b_1 - b_2}\right] > 0$. That is,

$$q^s \leq \frac{\zeta}{1 + \zeta} \equiv \bar{q}^s(k). \quad (2)$$

Observe further that $\zeta(k) > 0 \Rightarrow \bar{q}^s(k) \in (0, 1)$; that is, the upper bound on beliefs admits valid probabilities. We conclude that if $q^s \leq \bar{q}^s(k)$, then we can support a screening strategy for player 1 (observe that it does not depend on the discount factor).

It is possible that player 1 can himself make an offer he is sure will get rejected. Is making a non-serious offer a profitable deviation? Following such a deviation, beliefs remain the same and are only updated by the outcome of the battle. The game then begins with player 2's proposal, so we have to examine her equilibrium strategy first in order to determine what payoff player 1 should expect in that continuation game.

3.2 Player 2's Proposal Strategy

3.2.1 Separating Proposals: Signaling by the Weaker Type

Suppose 2_m makes an offer, $y(k)$, that player 1 accepts in equilibrium, and 2_s makes a non-serious offer that player 1 rejects. Following $y(k)$, player 1 will update that $q^m = 1$, and since he can always reject, any acceptable proposal must give him the expected payoff from fighting against 2_m and settling in the next period on $V_m^1(\cdot)$.¹ Hence,²

$$\pi - y(k) = (1 - \delta)b_1 + \delta [p_M V_m^1(k + 1) + (1 - p_M)V_m^1(k - 1)].$$

¹Strategies that support the full information payoff when $q^m = 1$ are analogous to case when $q^s = 1$. Since player 1's belief never changes even if anyone deviates from the strategy, he will always offer $V_m^1(k)$ and always accept any offer at least as good as $\pi - V_m^2(k)$. Clearly, 2_m and 2_w would have no incentive to delay. However, 2_s will not settle in any period, except possibly the first, given player 1's strategy. To see this, suppose it is player 1's turn to make an offer and he proposes $V_m^1(k)$. This is calibrated to make 2_m indifferent between accepting and delaying to obtain $V_m^2(\cdot)$ in the next period. The expected payoff for 2_s for rejecting and then mimicking 2_m is $(1 - \delta)b_2 + \delta [p_L V_s^2(k + 1) + (1 - p_L)V_s^2(k - 1)]$, which is strictly better than $\pi - V_m^1(k) = (1 - \delta)b_2 + \delta [p_M V_m^2(k + 1) + (1 - p_M)V_m^2(k - 1)]$ because $p_L < p_M$ and $V_m^2(k + 1) < V_m^2(k - 1)$. In other words, it cannot be an equilibrium strategy for 2_s to accept $V_m^1(k)$ in any period. An analogous argument shows that she will not propose $V_m^2(k)$ either. Hence, the subgame perfect strategy is to reject player 1's offer and make non-serious proposals in every period. Starting now in k , this means that player 2_s expects $W_s^2(k)$ if she does not settle immediately. If $V_m^2(k) \leq W_s^2(k)$, then she will not settle in k either. If, on the other hand, $V_m^2(k) > W_s^2(k)$, the strategy is a lot more involved. Roughly speaking, it requires that we start with an arbitrary k and suppose first 2_s would settle in both $k - 1$ and $k + 1$, in which case she will certainly fight in k . Then we proceed to analyze $k - 1$ assuming that she settles there but fights in k , and so on. The entire process is then repeated by supposing that 2_s would fight in both $k - 1$ and $k + 1$, and so on. This will construct a strategy that will involve fighting in some periods and settling in others. However, none of this makes any difference from player 1's perspective because the probability of having to face 2_s is 0, so his expectation is to settle for the full-information MPE payoff against 2_m .

²This is where the original proof went astray.

Since 2_m can always mimic 2_s and make an unacceptable proposal, the equilibrium proposal must be at least that good:

$$y(k) \geq (1 - \delta)b_2 + \delta [\pi - p_M V_s^1(k + 1) - (1 - p_M)V_s^1(k - 1)],$$

which gives us:

$$(1 - \delta)(\pi - b_1 - b_2) \geq \delta z_m,$$

where $z_m = p_M V_m^1(k + 1) + (1 - p_M)V_m^1(k - 1) - [p_M V_s^1(k + 1) + (1 - p_M)V_s^1(k - 1)] > 0$. This condition has a straightforward interpretation: z_m is the difference in expected payoffs from revealing one's type and mimicking the stronger player, that is, it is the "bonus" of delaying by one period; on the left-hand side is the remainder of the pie in the current period after a battle is fought. The only way 2_m would be unwilling to delay is when this remainder exceeds the future benefit of mimicry. Clearly, for high enough discount factors, this condition will be violated because $V_m^1(k) - V_s^1(k) > 0$ for all $\delta < 1$: intuitively, if 2_m is too patient, then delay is not sufficiently costly to deter mimicry. Hence, this introduces an upper bound on the discount factor. Let $\bar{\delta}(k)$ be such that 2_m will be willing to separate for all $0 \leq \delta \leq \bar{\delta}(k)$.

The next step is to establish whether 2_s will want to separate as well. Since she can always propose $y(k)$ as well and get it accepted, making a non-serious offer that reveals her type must be at least as good:

$$y(k) \leq (1 - \delta)b_2 + \delta [\pi - p_L V_s^1(k + 1) - (1 - p_L)V_s^1(k - 1)],$$

which simplifies to:

$$(1 - \delta)(\pi - b_1 - b_2) \leq \delta z_s,$$

where $z_s = p_M V_m^1(k + 1) + (1 - p_M)V_m^1(k - 1) - [p_L V_s^1(k + 1) + (1 - p_L)V_s^1(k - 1)] > 0$. As before, z_s is the bonus from having waited to convince player 1 that he faces 2_s . This bonus is greater for 2_s than it is for 2_m because since $p_L < p_M$, she is more likely to win the battle and end in the more attractive state $k - 1$. Note now that 2_s cannot be too impatient because for low enough delta, she would not want to delay agreement to signal her strength. Hence, this introduces a lower bound on the discount factor. Let $\underline{\delta}(k)$ be such that 2_s will be willing to separate for all $1 > \delta \geq \underline{\delta}(k)$.

Obviously, for such a solution to exist, it has to be the case that $z_m \leq \frac{(1 - \delta)(\pi - b_1 - b_2)}{\delta} \leq z_s$. A necessary condition for this to be satisfied is $z_m \leq z_s$, which is not difficult to demonstrate (in fact, it holds with strict inequality). However, deriving a sufficient condition is difficult because we must know the MPE payoffs and we don't have nice closed-form expressions for those that we can analyze. In fact, it's possible that no solution can be found given some combinations of exogenous variables. This a common problem with perfect sequential equilibria which may fail to exist if the discount factor is high enough. The culprit is the provision of an incentive for the weaker type to be willing to identify herself. Hence, while in principle it may be possible to find such a solution, it may be too restrictive in the sense that it may not work for a wide range of values of the parameters. I now turn to alternative strategies.

3.2.2 Pooling on a Non-Serious Proposal

Suppose that in equilibrium both 2_m and 2_s make non-serious offers that are rejected by player 1. Since the strategy is pooling, player 1's beliefs remain the same after he receives the offer, and only get updated by the outcome of the battle. Let $\hat{q}^s(k+1)$ denote the updated belief if he wins, and $\hat{q}^s(k-1)$ if he loses. Clearly, $\hat{q}^s(k+1) < q^s(k) < \hat{q}^s(k-1)$. There are several cases to examine:

1. $\hat{q}^s(k+1) \leq \bar{q}^s(k+1)$ and $\hat{q}^s(k-1) \leq \bar{q}^s(k-1)$; that is, player 1 will screen regardless of the outcome of the battle.
2. $\hat{q}^s(k+1) > \bar{q}^s(k+1)$ and $\hat{q}^s(k-1) > \bar{q}^s(k-1)$; that is, player 1 will not screen regardless of the outcome of the battle.
3. $\hat{q}^s(k+1) \leq \bar{q}^s(k+1)$ but $\hat{q}^s(k-1) > \bar{q}^s(k-1)$; that is, player 1 will screen only if he wins, and will not if he loses.
4. $\hat{q}^s(k+1) > \bar{q}^s(k+1)$ but $\hat{q}^s(k-1) \leq \bar{q}^s(k-1)$; that is, player 1 will screen only if he loses, and will not if he wins. This case cannot occur because it would imply $\hat{q}^s(k-1) \leq \bar{q}^s(k-1) < \hat{q}^s(k+1)$, which contradicts $\hat{q}^s(k-1) > \hat{q}^s(k+1)$.

Let's examine the first case first. Making an unacceptable demand results in a battle, and in the next period it leads to screening by player 1: that is, he offers $x(\cdot)$, and 2_m accepts it. Hence, the expected payoff to 2_m from making a non-serious demand is:

$$y^*(k) = (1 - \delta)b_2 + \delta [\pi - p_M x(k+1) - (1 - p_M)x(k-1)],$$

where $x(k)$ is defined by (1).

Since deviation is a zero-probability event, we need to assign beliefs to player 1 for that case. Since 2_s has an incentive to delay in the supposed equilibrium, it makes sense to assume that if player 1 observes an unexpected acceptable offer, he will conclude that it was 2_m that made it. (Note that this is a weaker requirement than the usual assumption which requires player 1 to believe the offer came from the weakest type 2_w .) Since the only profitable deviation is to make an offer that player 1 would accept immediately even though he believes he faces 2_m for sure, it follows that the proposal must satisfy:

$$\pi - y(k) \geq (1 - \delta)b_1 + \delta [p_M V_m^1(k+1) + (1 - p_M)V_m^1(k-1)],$$

or else player 1 will reject (having been convinced by the attempt that his opponent is 2_m). Deviation is not profitable if $y(k) \leq y^*(k)$, which after some algebraic manipulation reduces to:

$$\begin{aligned} & (1 - \delta)(\pi - b_1 - b_2) \\ & \leq \delta [p_M (V_m^1(k+1) - x(k+1)) + (1 - p_M) (V_m^1(k-1) - x(k-1))]. \end{aligned} \quad (3)$$

Noting now that

$$\begin{aligned} V_m^1(k) - x(k) &= \pi - (1 - \delta)b_2 - \delta [p_M V_m^2(k+1) + (1 - p_M)V_m^2(k-1)] - x(k) \\ &= p_M [V_s^2(k+1) - V_m^2(k+1)] + (1 - p_M) [V_s^2(k+1) - V_m^2(k-1)] \\ &> 0, \end{aligned}$$

where the inequality follows from $V_s^2(k) > V_m^2(k)$ because $p_L < p_M$, we conclude that the right-hand side of (3) is strictly positive and bounded away from 0 for any δ . This implies that the inequality can be satisfied for δ high enough. Let $\underline{\delta} < 1$ satisfy (3) with equality, so any $\delta > \underline{\delta}$ would also satisfy it. In other words, if 2_m is sufficiently patient he would wait to allow player 1 to screen her in the following period. This happens because the offer player 1 has to make to achieve this screening is calibrated such that 2_m would not want to pretend being 2_s , and as such it is actually better for 2_m than what she would have gotten in the full-information MPE. Correspondingly, since making an acceptable deviation today requires her to reveal her type, she can only get the (smaller) MPE payoff tomorrow instead of the one she gets when player 1 screens. Hence, if she is sufficiently patient, she would wait to obtain that bonus tomorrow. Observe that if 2_m is willing to wait, so is 2_s because she can always mimic 2_m in the next period but she is more likely to end in the more attractive state by winning the battle. Therefore, if δ is sufficiently high, the strategy of pooling on a non-serious demand by 2_m and 2_s is optimal conditional on player 1 screening in the next period regardless of the outcome of the ensuing battle.³

It is not difficult to see that if player 1 does not screen in the next period but instead offers $V_s^1(k)$, the full-information MPE proposal, that 2_s (and hence all other types) would accept, the payoff from pooling only increases for 2_m , thus increasing the incentive to delay by pooling with 2_s on an unacceptable proposal in the current period. Hence, the lower bound on the discount factor necessary to sustain this equilibrium will be lower. In other words, if δ can sustain pooling when player 1 screens regardless of the outcome of battle, it will certainly sustain pooling when he settles immediately either after victory or regardless of the battle outcome.

Finally, we need to specify the strategy for 2_w although in the proposed equilibrium with $q^w = 0$ player 1 never expects to play it (and this type can only appear in this continuation game by deviating from her equilibrium strategy). Since making an acceptable offer con-

³If, instead, we suppose that beliefs are unchanged by deviations, then the pooling strategy cannot be supported. To see this, note that if deviations from the proposed equilibrium strategy do not change player 1's beliefs, it follows that the only possibly profitable deviation is to offer enough to get player 1 to accept it immediately rather than reject in order to screen in the next period. Let $p = q^s p_L + (1 - q^s) p_M < p_M$ denote player 1's expectation that he will win the battle if he rejects an offer, where the inequality follows from $p_L < p_M$. (Observe that this is the equilibrium belief which will be unaffected by deviations.) Player 1's will only accept a (deviating) proposal, $y(k)$, if:

$$\pi - y(k) \geq (1 - \delta)b_1 + \delta [px(k + 1) + (1 - p)x(k - 1)].$$

His equilibrium strategy is to reject any offers that do not give him at least that much. This implies that the only possible deviation is for 2_m to offer the smallest $y(k)$ that would satisfy this condition with equality. For such a deviation to be unprofitable, it has to be the case that $y(k) \leq y^*(k)$, or:

$$\frac{(1 - \delta)(\pi - b_1 - b_2)}{\delta} \leq (p - p_M)(x(k + 1) - x(k - 1)),$$

which never holds because $p < p_M$ implies that the right-hand side is negative. Intuitively, if when we suppose that deviations are inconsequential in terms of player 1's beliefs, then a weaker opponent, 2_m , can take advantage of the relative pessimism and extract a deal today as if player 1 were still unsure of her type. However, since it is optimal for 2_s to delay to signal strength when player 1 makes an offer, it is odd to suppose that player 1 does not revise his beliefs downward if he suddenly sees an unexpected acceptable proposal: it can only have come from the weaker player.

vinces (wrongly) player 1 that it came from 2_m , this is the best 2_w can expect to get. If δ is high enough, even this type would prefer to pool in order to obtain the much higher payoff when player 1 screens in the following period. If, however, δ is lower, she would make the minimum acceptable proposal immediately, convince player 1 her type is 2_m , and end the game in the current period. For our equilibrium it is inconsequential which of these is the case since it is a zero-probability event and as such does not affect player 1's expected payoffs or the behavior of the other types of player 2.

3.2.3 Pooling on an Acceptable Proposal

Suppose now that $\delta < \underline{\delta}$, and so the strategies of pooling on a non-serious demand cannot be supported in equilibrium. Suppose further that it is even lower than the minimum discount factor that would make 2_m only willing to reveal her type by making an acceptable proposal. When players are that impatient, the only remaining alternative is for both 2_m and 2_s to make an offer that player 1 immediately accepts. Since these types pool on their proposal, player 1's beliefs will remain unchanged until after the battle. Let $p = q^s p_L + (1 - q^s) p_M$ denote player 1's expected probability of winning the battle. Again, suppose that he will screen in the next period no matter what that outcome. Player 1 will only accept a pooling proposal, $y^*(k)$, whenever:

$$\pi - y^*(k) \geq (1 - \delta)b_1 + \delta [px(k + 1) + (1 - p)x(k - 1)].$$

If player 2_s is willing to pool, so will 2_m , hence it is sufficient to establish that 2_s would not be willing to deviate by making an unexpected unacceptable offer. Assume that seeing such a demand convinces player 1 that his opponent is 2_s . Then, in the continuation game he will offer $V_s^1(\cdot)$ and all types would settle immediately. Deviating from the pooling strategy would then yield:

$$y(k) = (1 - \delta)b_2 + \delta [\pi - p_L V_s^1(k + 1) - (1 - p_L)V_s^1(k - 1)].$$

Player 2_s would not deviate even though this would convince player 1 of her type whenever $y(k) \leq y^*(k)$, or:

$$\begin{aligned} \delta [px(k + 1) + (1 - p)x(k - 1) - (p_L V_s^1(k + 1) + (1 - p_L)V_s^1(k - 1))] \\ < (1 - \delta)(\pi - b_1 - b_2). \end{aligned} \quad (4)$$

To see that the left-hand side of this inequality is strictly positive for any $\delta > 0$, observe that $p > p_L$ and $x(k) > V_s^1(k)$, where the latter follows from the fact that player 1 prefers to screen by offering $x(k)$ and risking rejection instead of settling immediately by offering $V_s^1(k)$. As δ goes to zero, the left-hand side converges to zero, while the right hand-side converges to the strictly positive $\pi - b_1 - b_2$. This now means that (4) will only be satisfied if δ is low enough. Let $\bar{\delta}$ solve this condition with equality: then it will hold for all $\delta < \bar{\delta}$ as well. Given the fact that 2_s is unwilling to delay, neither of the weaker types would have any incentive to attempt it either. This implies that their equilibrium strategy is to make the same demand.

What happens if player 1 is expected to settle on $V_s^1(\cdot)$ in the next period regardless of the outcome of battle? Preventing deviation then requires a slight modification of (4), as follows:

$$\delta(p - p_L) [V_s^1(k + 1) - V_s^1(k - 1)] < (1 - \delta)(\pi - b_1 - b_2),$$

which is satisfied for δ low enough. In fact, since player 1's expected payoff is lower (because he will be settling for less in the next period), it takes a smaller offer to satisfy him in the current period, making deviation even less tempting. In other words, if δ is low enough to prevent deviation when player 1 screens, it will certainly be enough to prevent deviation when he makes an acceptable offer.

3.3 Player 1 Does Not Screen in the Proposal Stage

Finally, suppose $q^s > \bar{q}^s(k)$, and so player 1 has no incentive to screen out 2_m : he is so convinced that his opponent is the strong type 2_s , that the expected payoff of offering slightly less on the small chance that she will turn out to be 2_m and accept it is not worth the risk of delay with the corresponding (relatively high) probability that he will lose the ensuing battle if he happens to be facing 2_s after all. Since the equilibrium offer must be acceptable to 2_s , we only need to examine potentially profitable deviations by the strong type: if she is willing to accept player 1's offer, so will all weaker types. Since only the strongest type would have an incentive to reject an equilibrium offer, assume that player 1's belief following rejection is $q^s = 1$, and so he will expect to settle on the full-information MPE, $V_s^2(\cdot)$, in the next period. If that's the case, the equilibrium offer must give 2_s at least:

$$\pi - x(k) \geq (1 - \delta)b_2 + \delta [p_L V_s^2(k + 1) + (1 - p_L)V_s^2(k - 1)],$$

which implies that $x(k) = V_s^1(k)$; that is, player 1 must make the full-information MPE offer as if he were playing 2_s for sure today. Hence, whenever $q^s < \bar{q}^s(k)$, his equilibrium strategy is to always propose $V_s^1(k)$ and reject any proposals worse than $\pi - V_s^2(k)$; if he observes an unexpected rejection, update to believe that $q^s = 1$, otherwise beliefs remain unchanged by deviations. All types of player 2 pool on demanding $V_s^2(k)$ with each type rejecting offers that are worse than waiting to settle in the next period (e.g., 2_m rejects any offer that does not give her at least $(1 - \delta)b_2 + \delta [p_M V_s^2(k + 1) + (1 - p_M)V_s^2(k - 1)]$). Of course, since player 1 never makes these lower offers, they are off the equilibrium path.

The last possibility is that player 1 makes a non-serious proposal that is sure to get rejected by both types, which means that his beliefs will only be updated by the battle outcome. If player 2 pools on a non-serious proposal in the next period, then the earliest player 1 can be expected to go screening is two periods in the future, and at that point he is faced with the same choice except maybe he will settle immediately instead of screening. In any case, this delay is costly and player 1 will strictly prefer to screen today.

3.4 Strategies

Table 1 shows the equilibrium strategies when $q^s \in (0, 1)$ and $q^m = 1 - q^s$, and when the discount factor is high enough to support pooling on non-serious demands by player 2 types.

Let $y_m(k) = \pi - (1 - \delta)b_1 - \delta [p_M V_s^1(k + 1) + (1 - p_M)V_s^1(k - 1)]$. Also, let $x_m(k) = \pi - (1 - \delta)b_2 - \delta [p_M V_s^2(k + 1) + (1 - p_M)V_s^2(k - 1)]$, and $x_w(k) = \pi - (1 - \delta)b_2 - \delta [p_H V_s^2(k + 1) + (1 - p_H)V_s^2(k - 1)]$. Note that there are many possible non-serious demands. The easiest is to demand the entire pie, which is what I have used in the table.

	$0 < q^s \leq \bar{q}^s(k)$	$1 > q^s > \bar{q}^s(k)$
1 proposes	$x(k)$	$V_s^1(k)$
1 accepts	$y \leq y_m(k)$	$y \leq V_s^2(k)$
2_w proposes	π	$V_s^2(k)$
2_w accepts	$x \leq x_w(k)$	$x \leq x_w(k)$
2_m proposes	π	$V_s^2(k)$
2_m accepts	$x \leq x(k)$	$x \leq x_m(k)$
2_s proposes	π	$V_s^2(k)$
2_s accepts	$x \leq V_s^1(k)$	$x \leq V_s^1(k)$
transitions	update to $q^m = 1$ if a proposal $y < \pi$ is observed, update to $q^s = 1$ if player 2 rejects $x(k)$, do not change beliefs after any other deviations, only update q^s with the outcome of battle by Bayes rule	update to $q^s = 1$ if out of equilibrium rejection occurs and otherwise update q^s with the outcome of battle only

Table 1: Equilibrium when $q^s \in (0, 1)$, $q^m = 1 - q^s$, and δ is sufficiently high to support pooling on non-serious demands.

Table 2 provides a numerical example to illustrate the equilibrium given by Table 1. Parameters are: $N = 10$, $\pi = 1$, $b_1 = 0.10$, $b_2 = 0.05$, $q^s = 0.10$, $p_L = 0.15$, and $p_M = 0.75$. For any starting state $0 < k < N$, the updated belief after player 1 winning a battle is $\hat{q}^s(k + 1) = 0.0217$, and after losing a battle, $\hat{q}^s(k - 1) = 0.2742$, provided player 2's action does not change his prior belief q^s . In this case, since $\hat{q}^s(k + 1) < \hat{q}^s(k - 1) < \bar{q}^s(k)$ for all k , it follows that player 1 would screen in any state k given his initial belief q^s (which will remain unchanged if the game starts with player 2's move because both her types pool on a non-serious demand). The column $U_1(x(k))$ lists player 1's expected payoff from making his screening offer $x(k)$; that is, it takes into account the probability that it will be rejected by 2_s .

To read the table, start with some state, say, $k = 6$. Suppose it is player 1's turn to make an offer. Screening with $x(k) = 0.35$ gives him an expected utility $U_1(x(k)) = 0.34 > 0.32 = V_s^1(k)$, which is what he would get from not screening and just offering 2_s 's full-information MPE payoff immediately (which would get accepted). If it is player 2's turn to make a proposal, type 2_m 's expected payoff from pooling with 2_s on a non-serious offer yields $y^*(k) = 0.56 > 0.26 = y(k)$, which is what she would get if she were to make an informative offer that player 1 accepts.

k	$V_s^1(k)$	$V_s^2(k)$	$\bar{q}^s(k)$	$x(k)$	$U_1(x(k))$	$y^*(k)$	$y(k)$
0	0.00	1.00	0.00	0.00	0.00	1.00	1.00
1	0.11	0.97	0.44	0.18	0.16	0.77	0.58
2	0.14	0.88	0.43	0.20	0.19	0.68	0.43
3	0.22	0.85	0.37	0.27	0.26	0.67	0.39
4	0.24	0.78	0.36	0.29	0.27	0.61	0.34
5	0.30	0.76	0.31	0.34	0.33	0.60	0.32
6	0.32	0.71	0.30	0.35	0.34	0.56	0.26
7	0.37	0.70	0.30	0.40	0.39	0.51	0.24
8	0.39	0.64	0.47	0.46	0.45	0.24	0.16
9	0.50	0.56	0.80	0.85	0.81	0.13	0.12
10	1.00	0.00	1.00	1.00	1.00	0.00	0.00

Table 2: Numerical Example for Equilibrium from Table 1.

4 Equilibrium when $0 < q^s, q^m, q^w < 1$

The solution is analogous to the case with only two types. (In fact, it can be extended to an arbitrary number of types provided there are sufficient number of forts to permit screening without hitting boundary conditions.) The solution essentially revolves around the incentives player 1 will have to screen out the weaker types, for which he must not be too pessimistic. He may choose to screen out q^w first, followed by q^m , or he may screen both q^w and q^m from q^s in one step without distinguishing which of the weaker types it is actually facing. The trade-off is between the costs of delay and the risk of having to fight a battle he will lose with a higher probability and obtaining slightly more on the bargaining table today (the familiar risk-return trade-off). Whether player 2 chooses to delay in order to signal her strength depends on the discount factor: when δ is very low, she pools on a common acceptable demand, when it is moderate the stronger types separate by making an unacceptable demand while the weaker ones make an acceptable one, and when it is high, all types pool on an unacceptable demand.

To make matters more precise, suppose in equilibrium player 1 screens out 2_w in the first period, then player 2 makes a non-serious proposal in the next, and player 1 screens out 2_m in the third period, with the game ending in the fourth period. In this equilibrium, the game from second period reduces to the case we've already examined: $q_w = 0$ and $q^s + q^m = 1$ with $q^s, q^m \in (0, 1)$. In this equilibrium the starting state, k_0 , is such that whenever a battle occurs, the continuation game equilibrium involves the screening we have already analyzed. Turning now to player 1's initiate choice, observe that since 2_w must be willing to accept the offer $x_0(k)$, it follows that in equilibrium it must give her at least as much as mimicking the behavior of the other types: that is, rejecting and making an unacceptable proposal in the next period, followed by mimicking 2_m and accepting player 1's screening offer in the third period. If she plays this strategy, 2_w can guarantee herself a payoff of $y_w(k)$ in the second period where

$$y_w(k) = (1 - \delta)b_2 + \delta [\pi - p_H x(k + 1) - (1 - p_H)x(k - 1)],$$

where $x(k)$ is defined by (1). This now means that her expected payoff from rejecting an

offer in the first period given player 1's beliefs that rejection signals $q^w = 0$, is:

$$(1 - \delta)b_2 + \delta [p_H y_w(k + 1) + (1 - p_H)y_w(k - 1)].$$

Since in equilibrium player 1 would never offer more than this absolute minimum, it follows that:

$$\pi - x_0(k) = (1 - \delta^2)b_2 + \delta^2\pi - \delta^2 z_w(k),$$

where $z_w(k) = p_H^2 x(k + 2) + 2p_H(1 - p_H)x(k) + (1 - p_H)^2 x(k - 2)$, which we can rewrite as:

$$x_0(k) = (1 - \delta^2)(\pi - b_2) + \delta^2 z_w(k) \quad (5)$$

Observe now that since the offer is calibrated to make 2_w indifferent between accepting it and mimicking 2_m 's strategy, it follows that the offer $x_0(k)$ is strictly less than 2_m 's expected payoff because 2_m is more likely to be in a more attractive state in two periods than 2_w (because $p_M < p_H$). Hence, 2_m is sure to reject this offer, and so will 2_s . What we need to show now is that player 1 will want to make this screening offer rather than satisfy all three types or both 2_w and 2_m .

If player 1 makes an offer that both 2_w and 2_m accept, his beliefs following rejection will be $q^s = 1$, in which case player 2 would demand $V_s^2(\cdot)$ in the next period, and player 1 will accept. If player 1 is willing to screen out 2_w , this should not be a profitable deviation. Recall that player 1's payoff from separating 2_m from 2_s in state k is:

$$U_1(k; q^s) = (1 - q^s)x(k) + q^s [(1 - \delta)b_1 + \delta (\pi - p_L V_s^2(k + 1) - (1 - p_L)V_s^2(k - 1))]$$

given that his beliefs are $q^s \in (0, 1)$ and $q^m = 1 - q^s$. Since player 2 is expected to make a non-serious offer, two battles will occur conditional on rejection of his initial offer. If the initial offer is rejected, player 1 will update that $q^w = 0$ and, by Bayes' rule,

$$q_1^s = \frac{q^s}{q^s + q^m} \quad \text{and} \quad q_1^m = \frac{q^m}{q^s + q^m} = 1 - q_1^s,$$

so his expected probability of winning the battle will be $p_1 = q_1^s p_L + (1 - q_1^s)p_M$. That is, he expects to be in state $k + 1$ with probability p_1 , and state $k - 1$ with probability $1 - p_1$ after the first battle. Given the outcome of the battle, he will revise his beliefs again. If he wins a battle (that is, if he is in state $k + 1$), he will believe that:

$$q_2^s(k + 1) = \frac{p_L q_1^s}{p_L q_1^s + p_M (1 - q_1^s)},$$

and if he loses the battle, he will believe that:

$$q_2^s(k - 1) = \frac{(1 - p_L)q_1^s}{(1 - p_L)q_1^s + (1 - p_M)(1 - q_1^s)}.$$

In either case, $q_2^m = 1 - q_2^s$. Since both 2_m and 2_s will make an unacceptable demand, player 2's proposal will not change player 1's beliefs, which means that after observing a proposal he will believe that the probability of winning a battle is $p_2 = p_L q_2^s + p_M (1 - q_2^s)$.

Finally, player 1 will update once more based on the outcome of the second battle before he makes his separating offer in the third period. If he wins, he will update that:

$$q_3^s(k+1) = \frac{p_L q_2^s(k)}{p_L q_2^s(k) + p_M (1 - q_2^s(k))},$$

and if he loses, he will update to believe that:

$$q_3^s(k-1) = \frac{(1 - p_L) q_2^s(k)}{(1 - p_L) q_2^s(k) + (1 - p_M) (1 - q_2^s(k))}.$$

Starting in k , we need to calculate the beliefs in the third period:

$$\begin{aligned} q_3^s(k+2) &= \frac{p_L q_2^s(k+1)}{p_L q_2^s(k+1) + p_M (1 - q_2^s(k+1))} = \frac{p_L^2 q_1^s}{p_L^2 q_1^s + p_M^2 (1 - q_1^s)} \\ q_3^s(k-2) &= \frac{(1 - p_L) q_2^s(k-1)}{(1 - p_L) q_2^s(k-1) + (1 - p_M) (1 - q_2^s(k-1))} \\ &= \frac{(1 - p_L)^2 q_1^s}{(1 - p_L)^2 q_1^s + (1 - p_M)^2 (1 - q_1^s)} \\ q_3^s(k) &= \frac{p_L q_2^s(k-1)}{p_L q_2^s(k-1) + p_M (1 - q_2^s(k-1))} \\ &= \frac{p_L (1 - p_L) q_1^s}{p_L (1 - p_L) q_1^s + p_M (1 - p_M) (1 - q_1^s)} \\ q_3^s(k) &= \frac{(1 - p_L) q_2^s(k+1)}{(1 - p_L) q_2^s(k+1) + (1 - p_M) (1 - q_2^s(k+1))} \\ &= \frac{p_L (1 - p_L) q_1^s}{p_L (1 - p_L) q_1^s + p_M (1 - p_M) (1 - q_1^s)}. \end{aligned}$$

Observe that, as expected, the beliefs $q_3^s(k)$ are the same whether player 1 won the first battle and lost the second or lost the first but won the second. Given these beliefs, we can now specify player 1's expected payoff from fighting for two periods and making a separating offer in the third:

$$\begin{aligned} w(k) = (1 - \delta) b_1 + \delta & \left[(1 - \delta) b_1 + \delta \left(p_1 p_2 (k+1) U_1(k+2; q_3^s(k+2)) \right. \right. \\ & + p_1 (1 - p_2 (k+1)) U_1(k; q_3^s(k)) \\ & + (1 - p_1) p_2 (k-1) U_1(k; q_3^s(k)) \\ & \left. \left. + (1 - p_1) (1 - p_2 (k-1)) U_1(k-2; q_3^s(k-2)) \right) \right], \end{aligned} \quad (6)$$

where we note that $p_2(k+1) \neq p_2(k-1)$ because of the updating from the outcome of

the first battle. In fact, simplification gives us:

$$p_2(k+1) = \frac{p_L^2 q_1^s + p_M^2 (1 - q_1^s)}{p_L q_1^s + p_M (1 - q_1^s)}$$

$$p_2(k-1) = \frac{p_L (1 - p_L) q_1^s + p_M (1 - p_M) (1 - q_1^s)}{(1 - p_L) q_1^s + (1 - p_M) (1 - q_1^s)}$$

Using these results, we can now simplify the probabilities as follows:

$$p_1 p_2(k+1) = p_L^2 q_1^s + p_M^2 (1 - q_1^s)$$

$$p_1 (1 - p_2(k+1)) = p_L (1 - p_L) q_1^s + p_M (1 - p_M) (1 - q_1^s)$$

and, using $1 - p_1 = (1 - p_L) q_1^s + (1 - p_M) (1 - q_1^s)$,

$$(1 - p_1) p_2(k-1) = p_L (1 - p_L) q_1^s + p_M (1 - p_M) (1 - q_1^s)$$

$$(1 - p_1) (1 - p_2(k-1)) = (1 - p_L)^2 q_1^s + (1 - p_M)^2 (1 - q_1^s).$$

These expressions are, of course, quite intuitive and it is worth noting that $p_1 (1 - p_2(k+1)) = (1 - p_1) p_2(k-1)$; that is, the probability of reaching the same state k after two battles (one victory and one loss) is the same no matter which of the two outcomes occurs first. We can rewrite:

$$U_1(k; q^s) = (1 - q^s) x(k) + q^s z(k)$$

where $z(k) = (1 - \delta) b_1 + \delta (\pi - p_L V_s^2(k+1) - (1 - p_L) V_s^2(k-1))$, and now simplify $w(k)$ from (6) by noting that:

$$p_1 p_2(k+1) U_1(k+2; q_3^s(k+2)) = p_L^2 q_1^s z(k+2) + p_M^2 (1 - q_1^s) x(k+2)$$

$$p_1 (1 - p_2(k+1)) U_1(k; q_3^s(k)) = (1 - p_1) p_2(k-1) U_1(k; q_3^s(k))$$

$$= p_L (1 - p_L) q_1^s z(k) + p_M (1 - p_M) (1 - q_1^s) x(k),$$

and

$$(1 - p_1) (1 - p_2(k-1)) U_1(k-2; q_3^s(k-2)) = (1 - p_L)^2 q_1^s z(k-2) + (1 - p_M)^2 (1 - q_1^s) x(k-2).$$

Putting everything together simplifies $w(k)$ from (6) as follows:

$$w(k) = (1 - \delta^2) b_1 + \delta^2 \left[p_L^2 q_1^s z(k+2) + p_M^2 (1 - q_1^s) x(k+2) \right. \\ \left. + 2 (p_L (1 - p_L) q_1^s z(k) + p_M (1 - p_M) (1 - q_1^s) x(k)) \right. \\ \left. + (1 - p_L)^2 q_1^s z(k-2) + (1 - p_M)^2 (1 - q_1^s) x(k-2) \right]$$

Since player 1's payoff from offering a separating $x_0(k)$ must be at least as good as making an offer only 2_s would reject, it follows that we have to show:

$$q^w x_0(k) + (1 - q^w) w(k) \geq (1 - q^s) x_d(k) + q^s z(k), \quad (7)$$

where $x_d(k)$ is the deviating offer that both 2_w and 2_m accept, and $z(k)$, as before, is the continuation payoff from fighting one battle and settling. Observe now that since $x_d(k)$ must induce 2_m 's acceptance given that player 1 will settle immediately next period (and if it does that, it will certainly induce 2_w 's acceptance as well), it follows that it must be equivalent to $x(k)$ from (1). That is,

$$x_d(k) = \pi - (1 - \delta)b_2 - \delta [p_M V_s^2(k + 1) + (1 - p_M)V_s^2(k - 1)].$$

By our assumption that $q^s(k) \leq \bar{q}^s(k)$, it follows that player 1 would prefer to separate 2_m from 2_s in the initial period: $(1 - q^s)x_d(k) + q^s z(k) \geq V_s^1(k)$. This implies that it is sufficient to show that he will be willing to separate 2_w from 2_m . That is, that (7) can be satisfied. Note now that $x_d(k) > z(k)$ simplifies to

$$(1 - \delta)(\pi - b_1 - b_2) > \delta(p_M - p_L) [V_s^2(k + 1) < V_s^2(k - 1)],$$

which always holds because the left-hand side is positive and the right-hand side is negative because $V_s^2(k + 1) < V_s^2(k - 1)$. Furthermore, since x_d is the offer that 2_m accepts rather than wait one period to settle as if she were 2_s , it follows that $x_d < \hat{x}_d$, where \hat{x}_d is the offer she would accept if she had to wait two periods for the same settlement. (That is, player 1 gets a larger share because player 2's expected payoff from the longer delay is lower). Since 2_w is more likely to end up in the less advantageous state after two battles compared to 2_m , it follows that player 1 can keep an even larger share if he were to offer \hat{x}_0 that would satisfy 2_w instead of her waiting two periods to settle as if she were 2_s ; that is, $\hat{x}_d < \hat{x}_0$. Finally, since getting the share 2_s is better than getting the share of 2_m , it follows that 2_w 's payoff from waiting two periods to obtain 2_m 's share is even lower, which means player 1 can obtain an even larger share: $x_0 > \hat{x}_0$. But now these inequalities imply that $x_0 > \hat{x}_0 > \hat{x}_d > x_d \Rightarrow x_0 > x_d$. Rewriting (7) in terms of q^w yields:

$$q^w \geq \frac{q^s z(k) + (1 - q^s)x_d(k) - w(k)}{x_0(k) - w(k)} \equiv \underline{q}^w(k). \quad (8)$$

Note now that $x_0(k) > x_d(k) > z(k)$ implies that $x_0(k) > q^s z(k) + (1 - q^s)x_d(k)$, which in turn means that the denominator is larger than the numerator. That is, $\underline{q}^w(k)$ is a valid probability. That is, there exist values $q_w \in [\underline{q}^w, 1)$ such that player 1 would prefer to screen out 2_w in the first period rather than make an offer than both 2_w and 2_m would accept. This also implies that he would not make an offer that all three types would accept. Observe that, as before, the condition that makes such separation profitable involves player 1's belief about the weak type: player 1 must think he is facing 2_w with high enough probability to induce him to risk delay and fighting by making an offer that only 2_w would accept.

A numerical example with $q^w = 0.60$, $q^m = 0.36$, $q^s = 0.04$ and $p_H = 0.95$ (with the rest of the parameters as in the example from the two-type case) produces the equilibrium in which player 1 screens out 2_w . Note that conditional on rejection, $q_1^s = 0.10$, which reduces the continuation game to the one shown in Table 2.

5 Conclusion

The original Proposition 2 only identified one form of the equilibrium in which player 1 screens out each type, while the strong player 2 signals by making unacceptable proposals.

As we have seen, this behavior can occur in equilibrium despite the mistake in the proof, although it appears to be more difficult to obtain than the two cases where player 2 pools either by making non-serious demands or unique acceptable ones. The substantive conclusions of the paper all follow as originally stated. In particular, fighting and bargaining reveal information, with the behavior at the negotiating table being more informative than noisy battlefield outcomes. The equilibrium has a screening flavor (as usual for these one-sided incomplete information games), and the more patient the players, the longer the delay. That is, the shadow of the future will tend to prolong war because players hold out for better deals. Beliefs will converge, as stated by the principle, and in fact the present result underlines the important claim that it is not necessary that all information gets revealed: as long as they are sufficiently close to make any further delay unprofitable, players will settle.

References

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