Feigning Weakness

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The Chinese Intervene in the Korean War

- Inchon/Pusan: North Korean army destroyed
- Should U.S. turn war of liberation into war of unification?
- U.S. does its best to ascertain PRC intent:
  - consultation with allies
  - reconnaissance overflights
  - military intelligence
  - careful reading of PRC press
  - observation of behavior in Beijing
  - PRC will not intervene
- U.S. estimate (late November): at most 70,000 “volunteers”
- Reality: over 300,000 crack troops in North Korea!
Some possible interpretations:

- PRC threat was credible, the U.S. irrationally dismissed it for cognitive, bureaucratic, psychological, or political reasons
- PRC threat was credible, the U.S. dismissed it because of PRC’s reputation for bluffing or military weakness
- PRC threat was not credible by design because it wanted to lure the U.S. into a war over North Korea
"It is not easy to explain why the Chinese entered North Korea so secretly and so suddenly. [...] They chose instead to launch a surprise attack, with stunning tactical advantages but no prospect of deterrence."

Thomas Schelling
A signal is credible if a weak actor is unwilling/unable to mimic it.

(a) Such signals must usually:
   - involve high risk of war
   - be very costly (immediately or later)

(b) Some mechanisms for credible signaling:
   - sinking costs (Fearon 1997)
   - tying hands (audience costs, Fearon 1994)
   - autonomous risk of war (Schelling 1960)
   - domestic political actors (Schultz 1998)
   - foreign political actors (Sartori 2002)
   - military mobilization (Slantchev 2005)
Costly Signaling vs. Chinese Theory

The costly signaling literature implies that:

- a strong actor never wants to pretend to be weak
- absence of costly signal is *prima facie* evidence of weakness

... but Sun Tzu’s principle states:

“If your opponent is of choleric temper, seek to irritate him. Pretend to be weak, that he may grow arrogant.”

Sun Tzu
Does not look like Sun Tzu was wrong when it comes to fighting:

- Warfare is costly, so always conserve effort
  - less effort if A believes B is weak
  - a strong B can take advantage of A’s belief

...but what about bargaining before fighting?
Implications for Crisis Bargaining

Contradictory incentives for a strong actor:

- during crisis: wants opponent to believe he’s strong (so she agrees to larger concessions)
- if negotiations break down: wants opponent to believe he’s weak (so she expends lower effort fighting)

Seems that the strong actor must somehow simultaneously signal strength and weakness.
How Can We Study This?

A minimalist model should have:

- bargaining in the shadow of power
- endogenous distribution of power in war-fighting
- fighting decisions depend on information gleaned from crisis

Model structure:

- Fearon’s original take-it-or-leave-it (TILI) crisis game
- war-fighting as costly probabilistic contest in efforts
- effort depends on beliefs that may be based on crisis behavior
Show that:

**a)** the strong actor benefits from opponent thinking him weak when war begins (not surprising, but nice)

**b)** this causes the strong actor to pretend to be weak during the crisis with positive probability
The Model: Structure & Payoffs

(a) Bargaining (Ultimatum) Phase:
- two risk-neutral players, 1 and 2
- bargain over division of benefit \([0, 1]\)
- player 1 makes TILI offer \((x, 1 - x), x \in [0, 1]\)
- if player 2 accepts, game ends (payoffs from shares)
- if player 2 rejects, war begins

(b) Contest (War) Phase:
- players simultaneously spend effort, \(m_i \geq 0\)
- victory determined by technology of war:

\[
\pi_i(m_1, m_2) = \begin{cases} 
\frac{m_i}{m_1 + m_2} & \text{if } m_1 + m_2 > 0 \\
1/2 & \text{otherwise.}
\end{cases}
\]

- payoff: \(\pi_i(m_1, m_2) - m_i/c_i\)
Two-sided incomplete information about costs of effort:

- each player knows own costs;
- player 1 believes player 2 is strong, $\bar{c}_2 > 0$, with probability $p$, and weak, $c_2 < \bar{c}_2$, with probability $1 - p$;
- player 2 believes player 1 is strong, $\bar{c}_1 > 0$, with probability $q$, and weak, $c_1 < \bar{c}_1$, with probability $1 - q$;
- beliefs are common knowledge

Assume strong type’s costs are at least somewhat lower than the costs of his weak opponent: $\bar{c}_j > \sqrt{c_j \bar{c}_i}$. 
Players optimize:

$$\max_{m_i} \left\{ \frac{m_i}{m_1 + m_2} - \frac{m_i}{c_i} \right\}$$

Equilibrium expected payoffs:

$$W_1 = \left( \frac{c_1}{c_1 + c_2} \right)^2 \quad \text{and} \quad W_2 = \left( \frac{c_2}{c_1 + c_2} \right)^2.$$
Basic setup from Fearon’s model the same:

$$W_1 + W_2 < 1,$$

so war is inefficient, so mutually acceptable peaceful division still exists under complete information.
The Informed Player

The informed player (1) optimizes:

$$\max_{m_1} \left\{ \frac{m_1}{m_1 + m_2} - \frac{m_1}{c_1} \right\}$$

This is enough for the following:

**Lemma**

*In equilibrium, either both types of the informed player participate in the contest (skirmish), or only the strong type does (war).*
Let $m_1(\bar{c}_1) > 0$ and $m_1(c_1) > 0$ be player 1’s type-contingent effort levels.

Player 2 has a (posterior) belief $\hat{q}$ and optimizes:

$$\max_{m_2} \left\{ \frac{\hat{q}m_2}{m_1(\bar{c}_1) + m_2} + \frac{(1 - \hat{q})m_2}{m_1(c_1) + m_2} - \frac{m_2}{c_2} \right\}$$

It is possible that $W_1(\hat{q}; \bar{c}_1) + W_2(\hat{q}; \bar{c}_2) > 1$. 
The Uninformed Player: War Equilibrium

Since $m_1(c_1) = 0$, player 2 optimizes:

$$\max_{m_2} \left\{ \frac{\hat{q} m_2}{m_1(\overline{c}_1) + m_2} + (1 - \hat{q}) - \frac{m_2}{c_2} \right\},$$

It is possible that $W_1(\hat{q}; \overline{c}_1) + W_2(\hat{q}; \overline{c}_2) > 1$. 
Sun Tzu’s Principle of Feigning Weakness

Lemma

The more confident player 2 gets that player 1 is strong, the more effort will she spend fighting him.

Lemma (Sun Tzu)

Player 1’s expected payoff from fighting decreases as player 2 gets more confident that he is strong.

(If player 2 thinks player 1 is likely to be weak, she expends less effort than she would have if she knew player 1 was strong. The strong player 1 benefits.)
Feint Equilibrium Structure

Feint equilibria have the following structure:

- player 1 makes either a low-value, low-risk demand, $x$, or a high-value high-risk demand $\overline{x} > x$ as follows:
  - weak type always demands $x$
  - strong type mixes between $x$ and $\overline{x}$
- weak player 2 accepts both demands
- strong player 2 accepts $x$ with positive probability and rejects $\overline{x}$ with certainty

In these equilibria, the strong player 1 pretends to be weak with positive probability during the crisis.
The Equilibrium Demands

Since only the strong player 2 rejects an equilibrium offer with positive probability, in the contest player 1 knows he faces the strong opponent and:

- after $x$: player 2 is unsure if player 1 is strong
  - strong player 2’s contest payoff: $W_2(\hat{q}; \bar{c}_2)$, so
  $$\bar{x} = 1 - W_2(\hat{q}(x); \bar{c}_2)$$

- after $\bar{x}$: player 2 knows player 1 is strong
  - if weak player 2 deviates and fights, $W'_2 < W_2(\bar{c}_1, c_2)$, so
    $$\bar{x} = 1 - W'_2$$
Demands, beliefs, and rejection risks

Range of Possible Low-Value Demands

For the strong player 2’s, the contest payoff

- $W_2 = W_2(c_1, \overline{c}_2)$ is the best
- $\overline{W}_2 = W_2(\overline{c}_1, \overline{c}_2)$ is the worst

Therefore, regardless of her beliefs, the strong player 2 will:

- accept any $1 - x > W_2 \Rightarrow x_1 = 1 - W_2$
- reject any $1 - x < W_2 \Rightarrow x_2 = 1 - \overline{W}_2$

The only belief-contingent responses are to $x \in [x_1, x_2]$

Lemma 5: there exists a unique $\hat{q} \in [0, 1]$ that satisfies $x = 1 - W_2(\hat{q}(x); \overline{c}_2)$, and this $\hat{q}$ is strictly increasing in $x$. 
Demands, beliefs, and rejection risks

Posterior Beliefs for Player 2
Strong Player 2’s Probability of Rejecting Demands

- If strong player 2 were to accept $x \in [x_1, \hat{x}_1]$ for sure, the strong player 1 will never make such low demands.
  - ⇒ posterior belief $\hat{q}(x) = 0$
  - ⇒ strong player 2 certain to reject such $x$
    - ⇒ strong player 1 tempted to deviate to $x$
      (player 2 erroneously believes he’s weak)
      - ⇒ posterior belief cannot be $\hat{q}(x) = 0!$

Strong player 1 must not deviate despite temptation to rationalize this belief. (Restrictions on player 1’s prior.)
Demands, beliefs, and rejection risks

Rejection of Demands $x \in [\hat{x}_1, x_2]$

- Because the strong player 2 is indifferent given $\hat{q}(x)$, any rejection probability is admissible.
- Because the strong player 1 is willing to feign weakness, he must be indifferent between the two demands.
- The rejection probability that satisfies this condition for any $x \in [\hat{x}_1, x_2]$, is $\tilde{r}_2(x)$.
- Because the weak player 1 must be willing to make the low-value demand, he should not want to make a zero-risk demand like $x_1$.
- Hence, the risk of rejection cannot be higher than $\bar{r}_2(x)$.
  $\Rightarrow$ Any $x$ supportable as low-value demand in feint equilibrium must imply $\tilde{r}_2(x) \leq \bar{r}_2(x)$
Demands, beliefs, and rejection risks

Probability of Rejection by Strong Player 2
Let $r_1(x)$ the probability with which the strong player 1 makes the low-value demand (feigns weakness).

Since it must induce $q^*(x)$, Bayes rule requires:

$$r_1^*(x) = \frac{q^*(x)(1-q)}{q(1 - q^*(x))},$$

which is a valid probability if $q^*(x) < q (p > \hat{p}_{\text{min}})$.

To ensure that the weak player 1 cannot profit by deviating to $x_2$, we find $\hat{x}_2 \in [\hat{x}_1, x_2]$ such that $U_1(x; c_1) \geq U_1(x_2; c_1)$.

The set $[\hat{x}_1, \hat{x}_2]$ exists (Lemma 8).
When the necessary conditions are satisfied, then any $x \in [\hat{x}_1, \hat{x}_2]$ can be supported in a perfect Bayesian equilibrium of the crisis bargaining game using the following strategies and beliefs:

- **The weak player 1** demands $x$. The strong player 1, demands $x$ with probability $r_1^*(x)$ and $\bar{x}$ with probability $1 - r_1^*(x)$.

- **The weak player 2** accepts $x \leq \bar{x}$, and rejects every other demand. The strong player 2 accepts $x \leq x_1$, rejects $x \in (x_1, \bar{x})$, accepts $x \in [\underline{x}, x_2]$ with probability $1 - r_2^*(x)$, and rejects $x > x_2$.

On and off the path, beliefs are given by $q^*(x)$. In the contest, players use the belief-contingent equilibrium strategies.
A Numerical Example

Let $c_1 = 1$, $\bar{c}_1 = 4$, $c_2 = 2$, $\bar{c}_2 = 5$, and $p = q = 0.7$, so $\bar{x} = 0.91$, and $x \in [0.41, 0.50]$. Take $\bar{x} = 0.45$, so $r_1^* = 0.30$, $r_2^* = 0.38$, and $q^* = 0.41$.

Feint benefit: $x$ ($\bar{x}$) is rejected with probability 0.27 (0.70), but the strong player 1’s war payoff is 0.31 (0.20). Pr(feint) = $qr_1^* \approx 0.21$. 
Costly Signaling and Feints

The costly signaling logic remains in the feint equilibria:

- the strong player 1 can only get $\overline{x}$ by running a larger risk of a costlier war.
- this discourages the weak from demanding $\overline{x}$ as well.
  $\Rightarrow$ Credible revelation of information requires costly signal.
Costly Signaling and Feints

With endogenous belief-contingent war payoffs:

- opponent’s beliefs matter in war
- incentives to manipulate these beliefs when one is strong
- strong reason not to reveal strength
  ⇒ strong player may foster *false optimism*
Overcoming Mutual Optimism…

Optimism and War:

- disagreement about how war will “play out”
- credible signaling: imperfect cure for mutual optimism

... but costly signaling is not the only problem!

A’s optimism may be deliberately induced by B:

⇒ information remains private because the only type of A with incentives to reveal it, does not want to
⇒ B cannot use lack of credible signal as evidence that A is necessarily weak
When A’s optimism is deliberately induced by B,

- A cannot use B’s costly signal to correct his own optimism because B’s signal may be a product of B’s false optimism that A cultivated

⇒ war provides the “stinging ice of reality” (Blainey)
Prior Beliefs for Player 1

The strong player 1’s payoff from the high-value demand $\bar{x}$:

$$U_1(\bar{x}; \bar{c}_1) = pW_1(\bar{c}_1, \bar{c}_2) + (1 - p)\bar{x},$$

Two necessary conditions for feint equilibria are:

$$U_1(\bar{x}; \bar{c}_1) \geq x_1 \iff p \leq p_{\text{max}}$$

(or else he would deviate to $x \leq x_1$ that is accepted for sure)

$$U_1(\bar{x}; \bar{c}_1) \leq x_2 \iff p \geq p_{\text{min}}$$

(or else he would not make a low-value low-risk demand)

The interval $[p_{\text{min}}, p_{\text{max}}]$ always exists.
Player 1’s Expected Payoffs

Let $r_2(x)$ be probability of strong player 2 rejecting $x \in [x_1, x_2]$.

The strong player 1’s payoff from the low-value demand:

$$U_1(x; \overline{c}_1) = pr_2(x) W_1(\hat{q}(x); \overline{c}_1) + (1 - pr_2(x))x,$$

The strong player 1’s payoff from the high-value demand $\overline{x}$:

$$U_1(\overline{x}; \overline{c}_1) = pW_1(\overline{c}_1, \overline{c}_2) + (1 - p)\overline{x} \equiv \hat{x}_1$$

(recall that $\overline{x}$ is fixed by the exogenous parameters)
Beliefs and Reactions Given Some $x \in [\hat{x}_1, x_2]$ 

Equilibrium Beliefs:

$$q^*(x) = \begin{cases} 
0 & \text{if } x < \hat{x} \\
\hat{q}(x) & \text{if } x \in [\hat{x}, x_2] \\
1 & \text{if } x > x_2.
\end{cases}$$

Equilibrium Rejection by Strong Player 2:

$$r_2^*(x) = \begin{cases} 
0 & \text{if } x < x_1 \\
1 & \text{if } x \in [x_1, x) \\
\tilde{r}_2(x) & \text{if } x \in [x, x_2] \\
1 & \text{if } x > x_2
\end{cases}$$
Putting It All Together