Supplement 1:
The Complete-Information Example

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This demonstrates the calculations for the subgame perfect equilibrium result discussed in my article "Military Coercion in Interstate Crises and the Price of Peace". We use the following parameters:

- $\lambda = 0.99$
- $v_1 = 0.60$, and $v_2 = 0.50$
- $c_1 = 0.20$, and $c_2 = 0.01$

With complete information, S_2 would never mobilize in equilibrium unless she is certain to attack if resisted. The choice then would be among fighting S_1, compelling him, or quitting—bluffing is not an option. S_1 will capitulate if $W_1^d(m_1, m_2) \leq -m_1$, or when

$$m_2 \geq \frac{m_1}{\lambda}\left(\frac{v_1}{c_1} - 1\right) \approx 2.02 \cdot m_1 \equiv \overline{m}_2(m_1).$$

Hence, if S_2 allocates $\overline{m}_2$ and gets S_1 to capitulate, her payoff would be:

$$EU_2^C(m_1) = 0.5 - 2.02m_1.$$

If S_2 allocates $m_2 < \overline{m}_2(m_1)$, then fighting is certain if S_1 has allocated m_1. The best S_2 could obtain from fighting is:

$$\begin{aligned} EU_2^W(m_1) &= W_2^a(m_1) \\ &= \frac{\lambda m_2^*(m_1) v_2}{m_1 + \lambda m_2^*(m_1)} - c_2 - m_2^*(m_1), \end{aligned}$$

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which, since $m_2^*(m_1, v_2) = \sqrt{\frac{m_1 v_2}{\lambda}} - \frac{m_1}{\lambda} \geq 0$, reduces to:

$$= v_2 - c_2 + \frac{m_1}{\lambda} - 2\sqrt{\frac{m_1 v_2}{\lambda}}$$
$$\approx 0.49 + 1.01 m_1 - 1.42\sqrt{m1}$$

Since S_2 would prefer fighting to compelling whenever:

$$EU_2^W(m_1) \geq EU_2^C(m_1) \Leftrightarrow m_1 \geq 0.226582.$$

That is, if S_1 allocates more than $m_1 \approx 0.23$, then S_2 would fight rather than bother compelling him to quit.

Since S_2 can always obtain $EU_2^Q(m_1) = 0$ by quitting immediately, she would prefer compelling to quitting whenever $EU_2^C(m_1) \geq EU_2^Q(m_1)$, or whenever $m_1 \leq 0.2475$. That is if S_1 allocates less than $m_1 \approx 0.25$, then S_2 would rather allocate the assured compellence level than quit.

Similarly, S_2 would prefer fighting to quitting whenever $EU_2^W(m_1) \geq EU_2^Q(m_1)$, or whenever $m_1 \leq 0.364893$. That is, if S_1 allocates less than $m_1 \approx 0.37$, then S_2 would rather fight than quit. Putting these results together, we obtain the following:

- $m_1 < 0.23 : EU_2^C > EU_2^W > EU_2^Q \Rightarrow S_2$ compels S_1 to quit;
- $0.23 \leq m_1 < 0.25 : EU_2^W > EU_2^C > EU_2^Q \Rightarrow S_2$ fights optimally;
- $0.25 \leq m_1 < 0.37 : EU_2^W > EU_2^Q > EU_2^C \Rightarrow S_2$ fights optimally;
- $m_1 \geq 0.37 : EU_2^Q > EU_2^W > EU_2^C \Rightarrow S_2$ quits.

This now gives us the subgame perfect solution to the continuation game as a function of S_1's initial choice. It is obvious that in equilibrium he will never allocate $m_1 < 0.23$, because he will have to capitulate for sure, and any such positive allocation is just a cost. Similarly, he would never allocate more than $m_1 = 0.37$ because S_2 is certain to quit for all such values, and so he would be paying more unnecessarily. Hence, S_1's choice boils down to $m_1 = 0.37$, which would lead to S_2's capitulation, or some $m_1 \in [.23, .37)$ that would lead to certain fighting.

If S_1 allocates the assured deterrence level, his payoff is $0.60 - 0.37 = 0.23 > 0$, so in equilibrium S_1 would never quit immediately.

What would he get if he allocates less than that and fights? For any such allocation, S_2 responds with her optimal fighting allocation $m_2^*(m_1)$, and so S_1's best possible fighting payoff is:

$$\max_{m_1} \left\{ W_1^d(m_1, m_2^*(m_1)) \right\} = \max_{m_1} \left\{ \frac{v_1}{v_2}\sqrt{\frac{m_1 v_2}{\lambda}} - c_1 - m_1 \right\}.$$

Taking the derivative and setting it equal to zero yields:

$$v_1 = 2\lambda\sqrt{\frac{m_1 v_2}{\lambda}},$$

and hence the solution is:

$$m_1^* = \frac{v_1^2}{4\lambda v_2} = 0.181818 \approx 0.18.$$

This means that the best S_1 can do if he is going to fight would be to allocate $m_1^* = 0.18$, in which case his expected payoff would be -0.02, that is, worse than quitting immediately. Of course, we know that for any $m_1 < 0.23$, no fighting will actually occur because S_2 would allocate at the assured compellence level, and so using $m_1 = 0.18$ yields S_1 an expected payoff of -0.18, even worse.

Therefore, optimal fighting is strictly worse than immediate quitting for S_1, but quitting is strictly worse than deterrence. This means that in the subgame perfect equilibrium, S_1 would allocate $m_1 = 0.37$, and S_2 would capitulate immediately. War never occurs with complete information between the adversaries with valuations $v_1 = 0.6$ and $v_2 = 0.5$.

Note how S_1's first move enables him to achieve deterrence even though his best war fighting payoff is worse than immediate capitulation! Why does this work? Because sinking the cost suddenly makes capitulation costlier than before: if S_2 resists, the new choice S_1 has is between quitting (which yields a payoff of -0.37, the sunk cost of mobilization), and fighting. The payoff from optimal fighting, assuming S_2 has mobilized at her optimal level, would be at least -0.05. Thus, S_1 has tied his hands by sinking the mobilization costs at the outset, and he will certainly fight if challenged. Because of S_1's rather high mobilization level, fighting is too painful for S_2 and so she capitulates.